VOLUME XXXIX APRIL, 1929 No. 2

THE MONIST

A CONCRETE INTERPRETATION OF SCHOPENHAUER'S NOTION OF THE WILL

STUDENTS of Schopenhauer generally admit and emphasize the Frankfort philosopher's uncommon contact with everyday reality in his thinking. The biographers and critics of Schopenhauer never fail to praise him for always basing his speculation on concrete experience instead of erecting a system of philosophy on the dangerous foundation of abstract theorizing. And it is only just to stress this practical turn of Schopenhauer's mind, because he himself insists again and again in his works that concrete or intuitive experience is far superior to abstraction. But when these critics expound Schopenhauer's philosophy, they usually forget the principle which they have recognized in theory, with the result that they encounter unnecessary difficulties in his thought. Thus certain objections are raised to Schopenhauer's theory of the will. But these objections, I believe, are based on a failure to bear in mind that, when Schopenhauer wrote about the will and its importance in life and the universe, he was not theorizing abstractly, but basing his remarks on concrete experience and on a first-hand contact with life.

Suppose we examine two of the objections which are usually raised against Schopenhauer's philosophy. Schopenhauer claimed that he had discovered the true nature of the universe, the thing-in-itself. Whereas Kant had

spoken vaguely of the ultimate reality as a mysterious *x,* Schopenhauer states positively that the thing-in-itself is *will,* and describes how we arrive at a knowledge of this will. The whole universe—the motion of the planets, the physical and chemical reactions of matter, plant and animal life, and human conduct itself—all these are governed by will. And we know that this is so, not as a result of logical reasoning, but by a mysterious intuition which borders on the miraculous.[1] Accordingly, Schopenhauer is accused of positing a metaphysical catch-word, which he personified into a Cosmic Will. This Cosmic Will, his critics claim, is as meaningless as Hegel's Cosmic Reason. Nietzsche, for example, accuses Schopenhauer of being a *Hinterweltler,* or a metaphysician, of inventing a reality which is unreal.[2] And M. Ernest Seillière, following in Nietzsche's footsteps, also speaks of an *au-delà,* namely, the world of will; an unreal, super-terrestrial sphere of activity, in every way the opposite of our daily existence which is purely phenomenal in the Kantian sense. Secondly, the objection is raised that Schopenhauer arrived at the notion of the Cosmic Will by an analogy from human conduct. And the analogy, it is claimed, is far from being exact.

Both these objections will disappear, I believe, if only we remember that Schopenhauer's theory of the will is based on concrete experience, not on abstract reasoning.

Schopenhauer analyzes human conduct and finds that reason plays a very minor role in life; or at best but a subordinate role. Life is irrational: people do not follow their reason, but act in obedience to some irrational, capricious instinct which is irreducible to the forms of knowledge because of its very capriciousness. This notion he expresses philosophically by stating that man is the incar-

[1] *Werke,* ed Deussen, I, p. 122; p. 478-9.

[2] *Zarathustra,* I, iii.

nation of will or caprice; that is his true nature. At bottom man is not a rational animal at all; on the contrary, he is an irrational, capricious, impulsive animal. He has no reasons for acting as he does, but only invents excuses to explain his actions as much as possible. And ultimately he can explain nothing, for aetiology, or the science of causal explanation, is very limited. For instance, I murder a man because I want his money; and I want his money because I want to buy a motor car or a diamond ring because the lustre of the stone pleases me and gives me a feeling of power. But I have really explained nothing; for why should a lustrous stone please me; why should gold and diamonds fascinate humanity any more than paste jewelry and ordinary pebbles? Is it not much more reasonable to look upon diamonds as Sir Thomas More regarded them in his *Utopia,* or Voltaire in his *Candide,*—with profound contempt? We cannot answer these questions. We must either believe that some system of reasoning, totally different from our own, can explain these puzzling problems; or else we must admit that they are not amenable to any system of reason at all. Religion accepts the former alternative and says that God knows all these mysteries. Schopenhauer, who was an atheist, concludes that they are not amenable to reason: we are governed by *will.*

The charge of positing a metaphysical catch-word, of hiding behind a *Hinterwelt,* in an *au-delà,* now vanishes. Far from being a *Hinterweltler,* Schopenhauer insists several times in his works that his philosophy deals with this world, and has no concern with any fictitious realm. He objects strenuously to such talk and calls it *Wolkenkukuksheim* (νεφελοκοκκυγία).[3] What Schopenhauer calls the world of will and the world of representation or intellect

[3] *Werke,* I, pp. 321-2; II, pp. 733-4.

is nothing more than a poetical expression of the duality of our nature; he means that every human being is, on the one hand, the creature of blind impulse or will; on the other, the possessor of reason, which, however, he uses only to help his will attain its desires. Accordingly, Schopenhauer usually speaks of the world *as* will, not of the world *of* will. The will is not an explanation of the phenomenal world, but a description of its true nature, a label which describes what happens in life, without seeking to explain why it happens thus. When Schopenhauer insists so frequently that we arrive at a knowledge of the will in a totally different way from all other knowledge, he is practically admitting that it is not knowledge at all, because the will is the irrational and unknowable as such. Only that which belongs to the realm of the intellect is amenable to the laws of knowledge; and the will, we must remember, is the very opposite of intellect.

Having established, at least to his own satisfaction, that man is the slave of will, Schopenhauer proceeds to examine the whole universe and finds that here, too, reason has fled and whirligig rules in his place. For we can no more explain why the planets should rotate, why the organic world should perform those physiological functions which it does perform, why things should be attracted to the centre of the earth,—we can no more explain these things than we can explain why human beings are pleased by diamonds. *De gustibus non est disputandum;* in other words, at the bottom of everything is will or caprice. And so by an analogy taken from human conduct, Schopenhauer arrives at a Cosmic Will. And by this he does not mean that there is some peculiar metaphysical creature called Cosmic Will running about in the universe, throwing pieces of itself upon individuals, who thereby become endowed with individual wills. He simply means that just

as human conduct is not guided by reason, but by caprice, so the whole universe is the embodiment of the same caprice or will. Regarded in this light, the analogy is very exact; and it may be noted here, in passing, that Anatole France in *Le Lys rouge* uses the same figure. In speaking of his heroine, who wishes to break away from her love for the hero, he remarks:

"C'était la fin. Elle le savait. Elle voulait rompre. Elle le voulait comme la pierre qui tombe veut tomber" . . . . etc.

I believe I am not reading theories of my own into Schopenhauer's philosophy. For in the final supplementary chapter to *Die Welt als Wille und Vorstellung*, which is entitled "Epiphilosophie," and in which he sums up his position on the main problems which his philosophical system attempts to solve, Schopenhauer states unequivocally that a *knowledge* of the *will* is a contradiction in terms. The laws of the intellect, what Schopenhauer calls the *Satz vom Grunde*, are applicable to the phenomenal world only; but they are unable to penetrate to the thing-in-itself, which is irrational.

"With our intellect, this mere tool of the will, we are everywhere striking upon insoluble problems, as against the walls of a prison. . . . It may at least be assumed as probable that not only for *us* is knowledge of all this (i. e., of the problems of ultimate reality—the will) impossible, but no such knowledge is possible at all, never and in no way; that these relations are not only relatively but absolutely insusceptible of investigation; that not only does no one know them, but that they are in themselves unknowable, because they do not enter into the form of knowledge at all. . . . The inner being-in-itself of things is not something which knows, not an intellect, but something without knowledge; knowledge is only added

as an accident, a means of assistance to the phenomenon of that inner being. . . . Here lies the reason why a perfect understanding of the existence, nature and origin of the world . . . is impossible."[4]

It is scarcely possible to state more clearly the limitations of knowledge and its inability to penetrate to the thing-in-itself than Schopenhauer has done in the above passage and in another which occurs in the second volume of *Parerga*.[5] Both these passages should be studied carefully, and then it will be seen that when Schopenhauer speaks of "knowledge of the will" all that he can mean is the recognition that our true nature is not at all accounted for by our intellectual knowledge, that the highest wisdom which we can attain with regard to the true nature of the universe is to know that we do not know.

One important consequence of this view of Schopenhauer's will is that it transforms the will into an essentially negative concept. If the will is the irrational and unknowable as such, it must be devoid of all positive characteristics, because everything that we *know* about the universe passes over into the realm of phenomena. It is really true, I think, that Schopenhauer's will is a negative notion. He really tells us nothing about it. He says the will is blind, which only means that it escapes the light of the intellect. He says it is free, which only means that it escapes the regulative control imposed upon phenomena by the intellect. He says it is one, which only means that it escapes all multiplicity; for multiplicity is a function of the intellect. All these are negative attributes; they are the opposites of the data which are furnished by the intellect. What shall we say, then, about Schopenhauer's claim that his notion of will was

[4] *Werke*, II, pp. 735-6.
[5] *Werke*, Vol. V, p. 98 ff.

more positive than Kant's mysterious x?[6] Schopenhauer is right. For Kant's vague notion of the thing-in-itself as a mysterious x (if we do not regard it simply as a limiting concept, which Schopenhauer certainly did not) admits the possible existence of some other order of reason, different or perhaps only more complete than ours. But Schopenhauer thinks that it is at least improbable that any order of reason should be able to fathom the nature of the universe, which is unreasonable as such. So much in theory; but practically he assumes that the attempt to reduce the true nature of the universe to some system of reason is not only improbable, but actually impossible.

This practical interpretation of Schopenhauer's view of the will applies not only to his metaphysics, but to his theory of art and to his ethics as well. Far from looking upon art as the imitation or the expression of life, Schopenhauer regards the aesthetic experience as the opposite of life; as an escape from it. In everyday life the intellect is the slave of the will; but during artistic contemplation this state of affairs is reversed, and the intellect becomes master of the will. Translated into everyday language this means that in our daily life we see things as we *want* to see them, while the artist or the work of art shows things as they really are. Schopenhauer thus arrives at what is apparently a paradox: far from looking upon art as an illusion of life, he regards it as truer than life. And he gives a very simple illustration of what he means. A man who is racing over the beautiful Rhineland, fleeing for his life from a pursuer, will have no eye for the natural scenery that surrounds him, because all his attention is directed to his means of escaping from his enemy. For such a man the bank of the Rhine will

[6] *Werke*, Vol. I, pp. 132-3.

be nothing but a straight line, and the bridges built across it nothing but lines running perpendicular to this straight line; his vision is one-sided, distorted, false. But the artist, who does not look upon the Rhineland as a means of escaping from a pursuer, who has no axe to grind, sees the Rhineland as it really is, in all its manifold beauty.[7] He has subdued his will or his personal desires, and is contemplating objects with his pure intellect. Schopenhauer's definition of genius as the preponderance of intellect over will, and of the artistic state as a condition of purely intellectual contemplation, may sound very metaphysical and abstract, especially as we are accustomed nowadays to connect art with emotion and to frown upon intellect in the realm of the Muses. But at bottom Schopenhauer only means that *he* has achieved truly aesthetic contemplation who has escaped from the ordinary way of regarding things as a means to satisfy our desires, and who views objects as ends in themselves; and this is concrete enough to be included in Mr. Clive Bell's decalogue of art.

Warning: When Schopenhauer speaks of the aesthetic experience as intellectual, he does not mean that art should be the product of logical concepts; nothing of the sort. Schopenhauer is constantly reiterating the uselessness of concepts in art.[8] By intellect, Schopenhauer understands not only logic, but what we would call intuition or imagination. He only uses the word intellect to exclude subjective interest or will.

All that has been said of art applies also to ethics; because the moral state is for Schopenhauer only a protracted condition of the aesthetic experience. In life as we ordinarily live it, we are the slaves of caprice or will. But the world as it should be is a state in which reason

[7] *Werke*, Vol. II, p. 434.
[8] *Werke*, Vol. I, pp. 276-8; II, p. 431.

has liberated itself from its bondage to the will and become supreme; the world as it should be is the life of reason. Life is unhappy because we are creatures of impulse, unable to follow the guidance of our intellects. But once we banish this will and put intellect in its place, we achieve a state of calm which is the most we can hope to obtain from life. Our misery is caused by our obedience to the behests of the will, by our affirmation of life; such happiness as we can attain is to be had only by the denial of the will to live. In plain speech Schopenhauer's ethic amounts to this: as long as we are the slaves of passion (the will), we are unhappy; we must renounce our passions and become contemplative philosophers, and only then can we taste what well-being there is in life. Hardly an original ethic, this; but that is entirely beside the point. It is an ethic which is based on a direct and concrete analysis of life; and it is only by recognizing how concrete are its foundations that we realize its practical value as a way of life.

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WHITEHEAD, CRITIC OF ABSTRACTIONS

(BEING THE STORY OF A PHILOSOPHER WHO STARTED WITH SCIENCE AND ENDED WITH METAPHYSICS.)

Introduction: Metaphysics and the Fall

WHAT was the exact nature of the Original Sin? No one seems to know for certain, but conjectures there have been a-plenty. Some have thought of the apple, and looked upon Adam as the first economic man, whose only fault was a healthy appetite. Others, like the Freudians, pointed knowingly at our fore-father's innocent nudity, and proclaimed that the disobedience was connected with the ominous fact of sex. Would it betray a professional prejudice to say that it was neither Adam's stomach, nor his wife, that got him into trouble, but his metaphysical lust? Eve, the apple, and the serpent are all there, to be sure, but they are not structurally essential to the picture. They are good as decorative effects, but not as necessary mechanisms. A curious Adam and a tree of knowledge would have done the trick.

My interpretation rests upon the Genesis story. But it would be true apart from it, and in spite of it. For human history from beginning to end has repeated the same mournful story, and lost Eden by wanting to know too much. When we come to the philosophers the moral is even more plain. Metaphysics has always been the forbidden fruit of philosophy, black-listed by common sense,

but even more desirable on that account. Every philosopher's story can be told in terms of that forbidden tree, whose mortal taste brought death and metaphysics into the world. And in most cases the play is a tragedy. Even John Dewey has had his moment of weakness, and his *Experience and Nature.*

Look at Whitehead now. See him starting out in the age of innocence of his first two books, scientist and natural philosopher pure and simple, fighting shy of metaphysics as prudishly as an old maid avoids sexy stories. Yet his very diligence and repeated repudiation of all metaphysics arouse our suspicion, and if we look at him more carefully, we can catch him taking long and eager sideway glances in the direction of the forbidden territory. The charm is too strong for him: willy-nilly he is driven towards it, and the very moment when he disclaims all connection with concepts of ultimate reality, he is laying the foundations for certain theories which will inevitably lead him there. It is a sad story from then on: we can see him drawing nearer and nearer, and before long he is munching greedily that apple which but a moment before he had proclaimed sour and crabby. If the story is old, and its end plain from the beginning, yet it does not lack in dramatic interest. Tragedy is the noblest of all the arts. And we might add that gossip is the most juicy of all diversions. There is nothing we enjoy so much as to watch a fellow-sinner's fall. So if the story is not too stale, and the moral too pointed, we may spend a few pleasant moments watching another good man go wrong.

If I keep the form of Adam's escapade in tracing the development of Whitehead's philosophy, it shall not be out of a pedantic desire to labor a far-fetched parable, but because the two fit together so well. Even a casual reader of Whitehead must realize that his work falls into three

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distinct periods: The "natural," the transitional, and the metaphysical. The first would coincide roughly with the first two books.[1] But I venture to say that even before he had finished those two books he had already entered the second stage, which must have occupied part of the time between his Tarner Lectures in 1919, and his Lowell Lectures in 1925. How long the transition was, we cannot tell. In his third book he has thrown all scruples to the wind, and has plunged into a bold and vigorous metaphysics, of which *Religion in the Making* is but a complement. His last book, *Symbolism,* by its subject-matter does not touch on metaphysics. I would place it earlier in the author's development. It seems to belong to the transition period. My task will not be that of a literary critic. I do not care whether my view of Whitehead's mental history is correct in all its details. All I want to do is to set forth his two main phases, the "natural" and the "metaphysical," and to try to show how the transition from the one to the other is not only intelligible under the circumstances, but indeed inevitable. If I can do this, and sketch the main outline in its separate stages and coherent development, I shall be satisfied.

I. *The Age of Innocence: Pre-Metaphysical. (Whitehead's Philosophy of Science)*

We might expect that Whitehead's first question would be, What is philosophy? But it is not. He could not have asked that question in 1919. Then he was not a philosopher. He was only a mathematician and a mathematical physicist, with a very narrow interest in the philosophy of

[1] Namely, *Principles of Natural Knowledge,* Cambridge, 1919, and *The Concept of Nature* (Tarner Lectures, 1919), Cambridge, 1920. Later works which deal directly with philosophy are: *Science and the Modern World* (Lowell Lectures, 1925), New York, 1926; *Religion in the Making* (Lowell Lectures, 1926), New York, 1926; *Symbolism, its Meaning and Effect* (Barbour-Page Lectures, 1927), New York, 1927.

science. The question as he asked it was more specific: What is the philosophy of science? Its answer was in the form of a further question: What are the ultimate concepts of science, and how are they related to the facts of our immediate experience? And no sooner was the question put but Whitehead proceeded to answer it in a double fashion, first, by a spirited critique of the current scientific concepts, and second, by a reformulation of those abstractions on the basis of a more adequate recognition of their relation to the concrete world. But before we look at the answer, let us make sure that we have understood the question. Just what does a philosophy of science involve?

It is significant that Whitehead starts with a realist's confession of faith. A philosophy of science cannot be bothered with metaphysics, but it cannot respectably avoid epistemology. For if we start by saying that the concepts of nature are in some way mental, that they owe their existence or their form to the nature of the mind, the whole discussion takes on a new coloring, and instead of talking about science and fact, we will be talking about scientists and metaphysical laws. So the way to start is by putting mind where it belongs, and making it plain at once that we positively refuse to be bullied by its insufferable presumption. So here goes: A philosophy of science consists of "homogeneous" thoughts about nature, i. e., of nature without mind except as a part of nature, as an event along with other events. What we want is "some account of the coherence of things perceptively known." (*Concept of Nature*, p. 29.) Knowledge, in other words, consists of relatedness, and we know when we grasp the relations of things among themselves. Mind is one of the relata in this system of relations, and knowledge is the

relation of this particular little block to the whole toy-castle of connected reality.

Now if knowledge consists in the structural relation of mind to the other natural entities that surround it, there is one factor that immediately becomes important, and that is what Whitehead calls "significance." It only means that the whole world does not leap in upon the mind, demanding admittance on the all-or-none basis, but waits patiently for its turn. Instead of saying in simple English that mind is selective in what it admits, he chooses the more roundabout way of making the distinction between the "discerned" and the "discernible," which slices up the world into what mind immediately attends to, and what it vaguely feels out of the corner of its eye, as a dim background. "The disclosure of an entity as a relatum without further specific discrimination of quality," (*Concept of Nature,* p. 51)—this is significance, and this means that mind can work with loose ends, and can know some things without knowing everything.

The "unexhaustiveness" of knowledge is what makes science possible, but it is also what makes it questionable. All scientific knowledge is either knowledge of the "discerned" or of the "discernible"—mostly of the latter. The discernible is dangerous territory. In it we seem to have left God's clear and obstinate light, and to have been transported to an artist's studio, with artificial light-arrangements, yielding a bizarre and unfamiliar world. The problem is how we can relate the artificiality of the studio to the easy naturalness of common sunlight—how to tie up the scientist's thin abstractions with the layman's solid experiences. We cannot ask to discern the discernible as discerned but we may we demand to see it as discernible, by having its connection with the discerned pointed out to us. So,

> "The primary task of a philosophy of natural science is to elucidate the concept of nature, considered as one complex fact for knowledge, to exhibit the fundamental entities and the fundamental relations between entities in terms of which all laws of nature have to be stated, and to secure that the entities and relations thus exhibited are adequate for the expression of all the relations between entities which occur in nature." (*The Concept of Nature*, p. 46.)

Whitehead now appears as the champion of the view that the concepts of science are descriptions of real characters of real events. But the question immediately arises: Which concepts? For it may very well be that there are certain scientific views which are so antiquated and so removed from concrete experience, as not to be worth defending. So a winnowing of scientific concepts seems necessary, if for no other reason, at least to eliminate false or partial statements. Whitehead's arch-enemy here is the orthodox world of physics as a series of instantaneous configurations of matter in a durationless space.

> "During the modern period the orthodox answer has invariably been couched in terms of Time (flowing equably in measurable lapses) and of Space (timeless, void of activity, euclidean), and of Material in space (such as matter, ether, or electricity)." (*Principles of Natural Knowledge*, p. 1.)

His criticisms of this mode of thought are the best introduction to his own constructive statement. Simply put, they are two, both levelled against the temporal and spatial disconnection of nature in which this view results:

1. If the world of science is a series of instantaneous snapshots, change becomes incomprehensible, and even the illusion of change, as given in the moving-picture machine, needs explanation. Motion needs time, and real time, not second-hand imitations of it in durationless instants:

> "Thus change is essentially the importation of the past and of the future into the immediate fact embodied in the durationless present instant." (*Principles of Natural Knowledge*, p. 2.)

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2. Space makes the world extended, and if extended, then disconnected, so that contact becomes inexplicable, and we are reduced to the billiard balls that push each other without touching. A moving-picture world that recurs at each instant, a world that is frozen stiff, and can neither move nor change, but somehow makes us think that it does—this is the state of affairs in which the traditional view leaves us.

Obviously there is something wrong here. There is a "fallacy of misplaced concreteness." It abstracts without knowing that it abstracts. To correct it we must return to the concrete world from which the abstraction was first made. What do we see there?

The ultimate fact of nature as unscientific, unphilosophical, unsophisticated perception sees it, is neither Time, nor Space, nor Material, nor a union of the three in worlds at an instant, but the flow of events. Open your eyes. The first thing you see are things happening. Start there. Events are the ultimate facts. And the moment you see events, you see their togetherness, a relatedness to each other, the most simple form of which is their overlapping or "extension."

Now Space and Time are not independent and self-sufficient entities. You never saw an instant, you will never touch a point. They are not immediately discerned, but they can be made discernible, if you consider the discerned relation of extension. Take two events of which one includes the other: A man eating strawberry shortcake. When he has done with his dessert, you know very well that the man contains the cake. But the cake contained strawberries, and the strawberries contained little seeds, and the seeds probably contained something else, and so forth. As you continue in this way indefinitely, you are approaching a point as a limit, and that point,

although not actually discerned by you, is at least discernible, since you can quite easily see how it is related to the discerned process of eating strawberry short-cake.

In exactly the same way you can arrive at an instant. Take another such "abstractive set"—a man who took a half-hour nap. Assuming that he slept through the whole of that half-hour, he must have slept through the half of it, and through the half of that, and so forth. If you go on halving, you will see that you are going straight for a tiny chunk of time which you will no longer be able to halve, which you could very definitely date (if your dating technique was sharp enough), but which would be so small as to be timeless. There you have your instant.[2]

Now are we any better off than the traditional view? Yes. We are left with points and instants of exactly the same sort as the old guard, but with this difference, that we know how we got them, while the old guard does not. But even so we would be in a bad fix. For points and instants as "abstractive elements" are no better than the same entities as ultimate facts. We would still have to face a world that is convulsively appearing and reappearing at successive instants. It is here that the special theory of relativity comes to our help. The world is made up not of one private time-system, but of many, and these time-systems are all objective and all different. And what is more, they obligingly intersect, and this intersection is the natural philosopher's boon. For the intersection of two time-systems (four-dimensional continuums) results in a three-dimensional solid; two three-dimensional solids

[2] Mr. V. F. Lenzen, of the University of California, thinks that he has disproved Whitehead's method of abstractive extension as a way of relating scientific concepts to the facts of concrete experience. His criticism seems to rest upon a misunderstanding of Whitehead, and I would have tried to answer it here, were it not that the same volume in which Mr. Lenzen's work appears, (*The Nature of Ideas,* University of California Publications, Vol. VIII, 1926), contains an adequate discussion of it, to which I have nothing to add. (See especially pp. 204-6.)

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intersect in a two-dimensional "level"; two of these give a one-dimensional "rect"; and two rects yield an "event-particle." Now we have an event-particle, which is important because it gives us the element of continuity, which we have missed before. It is a point-instant which fixes position in a four-dimensional aggregate, and so eliminates the jerkiness of the old world at an instant.

We spoke of different time-systems. Here is another concept which is only discernible, but not discerned. We have never seen two different time-systems side by side. What we have perceived, however, is equally to the point, and is sufficient to relate the different time-series of the special theory of relativity to the ordinary discerned world. And this is what Whitehead expresses by "duration" and "cogredience" with that duration. A duration is "a simultaneous whole of nature" (*Principles of Natural Knowledge,* p. 68); it is "the immediate general fact which is all nature now discernible" (*Concept of Nature,* p. 53.) A duration is a prolonged present, and cogredience is co-presence with that slice of nature included in the duration: "Cogredience is the preservation of unbroken quality of standpoint within the duration." (*Concept of Nature,* p. 110.) The "simultaneity" of the duration does not mean instantaneousness. So long as the sense of the present, of the "here" continues uninterrupted, cogredience with that duration is maintained. But let there come a change of perspective, and one duration snaps into another: cogredience has shifted, and with the shift the percipient event has taken a dive from one time-system into another. In this way time-systems are made discernible by being correlated to spatial perspectives, so that every duration, every present-here defines a time-system.

But this becomes even more important when we remember the paradoxes of relativity in regard to rest and

motion, which have made the bewildered layman wonder whether the elevator is going up the Tribune Tower, or the Tribune Tower sinking past the elevator. Is there any unequivocal meaning for rest and motion? Cogredience pulled us out of a ditch in showing how different time-systems are discernible; it will be just as useful now, in giving a definite meaning for motion and rest within one time-system. "Cogredience is the relation of an absolute position within a duration." (*Principles of Natural Knowledge,* p. 71.) In other words, cogredience defines absolute rest for the percipient:

> "The sense of rest helps the integration of durations into a prolonged present, and the sense of motion differentiates nature into a succession of shortened durations." (*Concept of Nature,* p. 109.)

The paradox still remains that what is rest in one time-system may be motion in another. But this need not disturb us. All we have to do is to fix our vantage-point, and then determine rest or motion, which shall be absolute within that time-system.

There is still another problem which is the bug-bear of every philosophy of science, and which Whitehead's four-dimensional geometry helps him solve: the question of congruence. Now congruence is presupposed in all measurement and, therefore, cannot be established by resort to that method. How do you know your yard-stick remains the same at different times and positions? But measurement is something fundamental to all science, and a philosophy of nature cannot avoid considering it. Congruence is immediately discerned in all recognition of simple permanences. It is discernible only through another excursion into four-dimensional geometry. It is established by a consideration of the three types of parallelograms (two pairs of rects, a pair of rects with a pair of point-tracks, and two pairs of point-tracks) which arise in two time-

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systems moving relatively to each other. Taking this symmetry of motion (i.e., the fact that, "Relative velocities are equal and opposite"—*Concept of Nature,* p. 130) together with the theory of perpendicularity, Whitehead extends congruence from one to all time-systems. This gives him the formula for transformations of measurements from one system to another, which for practical purposes is equivalent to the Lorentz transformations and the special theory, but is not based on such an arbitrary fact as the velocity of light:

> "It simply marks the fact that our congruence determination embraces both times and spaces in one universal system, and therefore if two arbitrary units are chosen, one for all spaces and one for all times, their ratio will be a velocity which is a fundamental property of nature expressing the fact that times and spaces are really comparable." (*Concept of Nature,* p. 193.)

There are two more concepts which must be taken up before completing Whitehead's philosophy of science: His theory of objects and his theory of organisms. Objects depend on recognition; they represent the permanent, the timeless, and recurring aspects of nature. An event can never be repeated. Its very essence consists in its fleeting individuality, which trickles into the stream of the universal flow and passes for ever, never to return. But objects are exactly those things which persist. How could we recognize them, if they did not? And note that recognition does not necessarily involve an intellectual judgment: "it is in its essence merely sense-awareness in its capacity of positing before us factors in nature which do not pass." (*Concept of Nature,* p. 125.) Objects (corresponding to Santayana's essences) are implied in all significant experience. They lie at the basis of all descriptions of the characters of events. Without them we would be left in a chaotic situation, where all things flow, and we do not even know that they flow.

We may distinguish several kinds of objects, starting in our regular way from the discerned to the discernible. Sense-objects are the termini of pure sensations. A patch of color, a floating smell, a burst of sound—any one of these are sense-objects. The distinctiveness of sense-objects, which differentiates them from all other objects, lies in their full-flavored individuality, since they are perceived in themselves and not as relations between other entities. The primitiveness of sense-objects is not shared by perceptual objects, which are late-comers in the field of knowledge. They arise out of the supplementing of one sense-object immediately discerned by the fund of previous experience, which now enters into the perceptual object as discernible. A perceptual object may be a delusion. It will be a "physical object"

> ". . . when (1) its situation is an active conditioning event for the ingression of any of its component sense-objects, and
> (2) the same event can be the situation of the perceptual object for an indefinite number of possible percipient events." (*Concept of Nature,* p. 156.)

A physical object which has been shorn of all those elements which are transient and dependent upon their relation to the mind becomes a "scientific object" and now definitely enters the province of science. A scientific object embodies the permanent characters of events. It is not discerned but discernible, since it is not an event at all, but only a refinement from the facts of immediate experience.

Organisms are a special class of scientific objects. They involve two essential elements: patterns and rhythms. Patterns are what some logicians call "continuous" or "internally related" wholes. They are structural and integral, not additive, wholes. The whole is more than the sum of the parts. "The quality of the whole is the rela-

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tionship between its parts, and the relation between the parts is the quality of the whole." (*Principles of Natural Knowledge,* p. 195.) Rhythm expresses the fact of the life-cycle of the organism, whose existence is vibratory in character, and requires a certain minimum of time in which it can be itself. In this way, "There is no such thing as iron at an instant." (*Ibid.,* p. 23.) It needs a minimum of time in which the electrons can rotate around the core of positive electricity and manifest those constants which constitute the nature of iron. This concept of organisms, intended primarily for biology, is already applied to inorganic matter in the *Principles.* It is not a main issue of the book, and is completely omitted in the *Concept of Nature,* but it takes one's breath away to think of the possibility of extending this concept to cover the whole field of physical nature.

Here our short sketch of Whitehead's natural philosophy must come to an end. Some rather essential elements which belong here have been omitted because they can be treated more conveniently later. This discussion is not complete. It only touches upon the high-spots. And even those high-spots are not considered exhaustively. Whitehead's answer is suggested rather than repeated, and in the case where geometrical demonstration is involved, I have incorporated the results without reproducing the deduction.

Yet even this summary must have made a few things clear. Whitehead's aim was ambitious: to relate scientific concepts to concrete experience. His results are not exhaustive, by any means. He has only occupied himself with a limited number of concepts, and that in a rather special way. Yet we cannot deny that within that narrow field he met with a certain measure of success. His criticism of nature-at-an-instant of old-fashioned physics is

well-taken and valid throughout. His own conception of nature as one complex of events with their various interrelations; his fundamental distinction between the discerned and the discernible; his derivation of points and instants as abstractive elements from the general relation of extension; his establishing of absolute rest in relative time-systems; his theory of congruence which yields a means of transformation from one perspective to another without relying entirely on such an arbitrary velocity as that of light; and finally his working out of the theory of floating objects which can be objective without being absolute, and correspond to real characters of events without hypostatizing them into new things—all these would mark out Whitehead's work as brilliant and significant. He states the task of a philosophy of science with admirable clarity and precision, and then proceeds to exemplify the method to be followed in achieving it by a constructive contribution whose only lack is in the way of completeness.

II. *The Temptation: Induction*

So far Whitehead was seen as a pure natural philosopher. He has definitely repudiated the problem of ultimate reality and contented himself with giving a consistent and restrained account of the relation of scientific concepts to actual experience. Yet in this very initial philosophy we can see a potential metaphysics. A daring and elaborate philosophy of life has its germs in this cautious philosophy of nature. All it needs is to be further unfolded. Yet, I venture to say, no material changes have to be made, no wholesale additions or alterations. The giddy metaphysical heights of *Science and the Modern World* and *Religion in the Making* are not due to a change of itinerary, but are merely the result of the following out of plans of an excursion whose first stages were made in the *Principles*

and the *Concept of Nature.* This shall be the thesis of this section.

Our study of the transition from the scientific to the metaphysical stage is the most interesting and also the most difficult part of this paper. Its absorbing interest lies not only in its "human" and biographical importance as the unfolding of a mind, but also in its exemplification of the more objective and universal problem of the relation of a philosophy of science to a philosophy of life. The difficulty of the enterprise is due to the fact that a great deal of our representation must be conjectural. We have no record of the transitional stage. All we have are the two stages, each more or less complete and self-sufficient, with an intervening period of six years of which we know nothing. What we have to do is to take the first period and see its unsolved problems, and then to take the last three works and note how they are related to the first two as a continuation and completion.

We find a clue in the quotation that appears in *Science and the Modern World,* and which is the clearest statement of the three major problems with which Whitehead was faced after his pre-metaphysical stage. Trying to justify the two chapters on Abstraction and God, he says:

> "Their justification is to be sought, (1) in our direct knowledge of the actual occasions which compose our immediate experience, and (2) in their success as forming a basis for harmonising our systematised accounts of various types of experience, and (3) in their success as providing the concepts in terms of which an epistemology can be framed." (p. 227.)

The problems as he has stated them are three: Induction, the Interrelation of Different Sets of Abstractions, and the Metaphysics of a Theory of Knowledge. The first, namely the problem of Induction, forms the link from the natural to the metaphysical period: it is the fruit of the forbidden

tree, which, if tampered with, will bring all the consequences of a fall into a full state of metaphysical sin.

The problem of induction has been the eternal puzzle of realistic philosophy. Beyond any question it is the hardest nut it has ever attempted to crack. It has broken many a sound tooth over it, but no one can say that it has so much as scratched it yet. And the worst of it is that if realism is to maintain its own self-respect as a creditable philosophy of science, it must crack it at all costs. For realism has banked its whole fortune on the observation of facts. Its main weapon against idealistic and pragmatic interpretations has been the eternal cry of "Facts!" And now it appears that it is getting facts with a vengeance. It has been left with its beloved facts and facts alone, surrounded by facts on all side, so that it is almost smothered by them. For unless it can find some explanation for induction, however desperate, it cannot stir a step beyond the immediate facts of immediate observation. So far from being able to generalize from one observation to a universal law, it cannot even move from one fact to many similar facts, or even pass from a present observation to an eternally determined fact.

The realistic attempts to give some rational explanation for induction are praiseworthy only for their humility. Russell thinks that probability is all he needs, and he believes to have established that. But if probability itself was empirically proved, it must presuppose induction, and hence cannot be used to explain it. If, on the other hand, probability is not taken from experience, but is a law of the mind and the precondition of experience, then why not say outright that induction is justified as a necessity of the mind, and be done with it? In either case, probability has not helped the realists much, except to sober their claims and temper the disappointment of their failure, not by

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adding to their achievement, but by reducing their expectations.

A. *Induction, as Universality of Connection.* Whitehead in particular must have felt keenly this deficiency of the realistic position which he was advocating. His whole method is based upon an appeal to naive experience, and any appeal to experience raises the problem of induction. Curiously enough the problem is never explicitly stated in either of his first two books. Yet consciously or unconsciously he is continually rubbing up against it.

One way in which he meets it is in the whole question of the continuity of events, and the universal relatedness of things. The importance of this phase of the problem becomes clear when we remember that this is precisely the method which the logicians of the German school of critical idealism use to establish induction. The process, as exemplified by Cassirer, is painfully simple: Grant once that there is a universal chain of necessary connection; that every event is the point of intersection of a series of laws; and your problem of induction is settled. All you have to do now, is to subsume that particular occurrence under one of these universal relations. The internality of relations is the simplest way to settle the difficulties of induction. From the strictly realistic point of view this is the forbidden fruit, and this is the temptation which Whitehead has now to face.

He is attracted to it from the very first. Any scientist would be. On the purely physical side he gives it expression in the two doctrines of the "ether of events" and the concept of the "field." The first stands for the fact that "something is going on everywhere and always": "the doctrine of transmission should be construed as a doctrine of the coextensiveness of events with space and time and of their reciprocal interaction." (*Principles,* p. 25.) The

concept of the field expresses the same idea from the standpoint of electromagnetism: "The field of an electron extends through all time, and all space, each event bearing a certain character expressed by its relation to the electron." (*Ibid.*, p. 96.) Does that mean the importation of the internality of relations wholesale into science? Whitehead saves himself from such an extreme position by differentiating the field into "occupied" and "unoccupied" events. The "occupied" part of the field is the situation of the object, arrived at by the usual method of extensive abstraction. It is the nucleus which shades off into an indefinitely extensive field, continually thinning out as it goes farther and farther away from home. So, on the one hand, nature is conceived as a net of interlocked events, and scientific research is given the assurance that its findings can never be merely local, but shall have universal significance. On the other hand, intensive investigation of a narrow field is made possible by the proviso that while the field of events is co-extensive with the whole of nature, it is best studied in its "occupied" events, which are the most essential part of its general nature.

Exactly the same thing is done with the more general problem of perception. On the one hand, he insists that all perception includes the whole of nature, if not as discerned, at least as discernible, if not as actually before us, at least as a half-felt background. "The whole which is present for discrimination is posited in sense-awareness as necessary for the discriminated parts." (*Concept of Nature,* p. 142.) On the other hand, he meets Bertrand Russell's objection that in that way no one would be able to know anything without knowing everything, by bringing in his doctrine of "significance": "The so-called properties of things can always be expressed as their relations to other things unspecified." (*Principles,* p. 12) This is

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the "unexhaustive" character of knowledge: "nature as perceived always has a ragged edge." (*Concept of Nature,* p. 50.) So we can, and in fact do, know some things without knowing everything, even though the whole is involved in every perception of a part.

His theory of scientific objects and scientific procedure repeats the same story. The "true purpose of an intellectual explanation in the sphere of natural philosophy," he says, "is to exhibit the interconnections of nature, and to show that one set of ingredients in nature requires for the exhibition of its character the presence of the other set of ingredients." (*Concept of Nature,* p. 141.) He repudiates the "accidental" view of nature: "nothing in nature could be what it is except as an ingredient in nature as it is." (*Ibid.,* 142.) Scientific objects are expressions of the permanent relations between events that hold throughout all nature. Thus every scientific object is a witness to this network of relations in the stream of events. The concept of isolated objects and isolated systems is only a matter of technical expediency, and does not express substantial independence, but only practical separation for purposes of specialized consideration.

Let us stop now, and feel the author's metaphysical pulse. The least metaphysical that one can be about induction is to think nothing about it, and not assume universal connections unless one finds them. You can take it as a working-principle that the thing you are now studying may be connected with other things, but you may be satisfied with a provisional agnosticism, until you are convinced one way or the other by subsequent observations. Apart from the mob of philosophically illiterate scientists who dogmatically assume that induction is legitimate because they use it, most thoughtful investigators, who see

the difficulties involved in the process, are content with such a provisional agnosticism.

Whitehead goes beyond this position in two ways. First, he assumes that there is a universality of necessary connection; accident is banished from nature. Then, he mitigates the rigor of his first statement by upholding the possibility of focussing attention on a small field, and neglecting the existence of the rest of the world except as a background. So far, there is nothing metaphysical in these statements. They can be taken as mere expressions of the assumptions that scientific procedure unconsciously makes. Yet the danger-line is only a step away. The moment Whitehead takes these as more than scientific "rule o' thumb," and attempts to give them a philosophical justification, his treatment cannot help containing something of a metaphysical character. The partition that divides universal relatedness from universal prehension is very thin indeed.

B. *Induction, as the Order of Nature.* So far we have been dealing with only one aspect of induction, which is perhaps not the most essential and certainly not its most obvious phase. We have still to consider induction in its most primitive form as the uniformity of nature. Whitehead never tackles this problem directly. He stumbles across it when dealing with congruence. All measurement is a form of induction. It makes the presuppositions of all induction, namely the irrelevance of the particular date and place of the observation to the validity of the determination. The foot-rule does not change when moved from one position to another, and the passage of time leaves it identical with itself. If my understanding of Whitehead's treatment of the problem is adequate, it appears that his geometrical justification of congruence concerns mainly the transformation of measurements from one time-system

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to another. But that itself is only a later problem, and is directly based on the assumption of the essential timelessness of space-time measurements in any one time-system. Relative determinations are supposed to be absolute within their relativity, and once accepted, to hold good for all observers and all times. Here the fundamental issue is dodged. Every step of the argument presupposes induction in its rudimentary form: Once true, true always and everywhere. The very formula of transformation, in so far as it depends on experience either for its determination or for its verification, must stand squarely upon induction.

The need of a rational justification of induction does not seem to have occured to Whitehead till after his first two books. We meet it for the first time in *Science and the Modern World,* where it appears fully thought out and boldly stated:

> "Induction presupposes metaphysics. It rests upon an antecedent rationalism." (p. 65.)
>
> "Either there is something about the immediate occasion which affords knowledge of the past and future, or we are reduced to utter scepticism as to memory and induction." (p. 64.)

The whole metaphysical system set forth in this book is such a rationalization of each actual occasion, so as to make induction possible. A sketch of the metaphysical position will be given later. The only part that we need notice here is that which concerns the "order of nature." With great acuteness Whitehead resolves the problem of induction into the problem of time. If we start with time as mere succession, we shall be forced to Hume's conclusion that there is no necessary connection of past to future. But what induction presupposes is just this "irreversible relationship of settled past to derivative future."[3] And it

[3] *Symbolism,* p. 35. I have turned to this book, in preference to *Science and the Modern World,* because of its greater clearness and concentrated attention on this particular problem.

is not only the universal feeling of the order of the world which proclaims that history is not chance but destiny, and that everything which is, is necessarily and inescapably so; every common 'stubborn' fact points to the same conclusion. Look at time as a durée réelle, where the whole past flows into the present and makes it "heavy with the contact of things gone by"; forget about time as the succession of instantaneous knife-edge presents; and your problem of induction is solved. What is true now will be true always, not because mental past is preserved in mental present, but because the actual past never dies, but lives on eternally in the ever-growing present.

Could he have stopped here? He has reduced induction to the self-consistency of nature, where the present receives the past as its inescapable heritage. But how has that helped induction? How is it different from the bare assumption of the uniformity of nature? Only in this respect: that the postulate of uniformity is rationalized into conformity, and given a wider explanation. Nature is conceived as a creative advance, as an organism, where the whole operates as a unit. There is an interesting passage in the *Concept of Nature* which expresses the earliest appearance of this view. It comes like a final exclamation-point at the end of the chapter on Time. It is not dogmatic and definite, but musing and awed before the mystery of time. And wonder is always to be regarded with suspicion, for mystery always suggests a metaphysics, and he who wonders already knows too much:

> "The past and the future meet and mingle in the ill-defined present. The passage of nature has no narrow ledge of definite instantaneous present within which to operate. Its operative presence which is now urging nature forward must be sought for throughout the whole, in the remotest past as well as in the narrowest breadth of any present duration. Perhaps also in the unrealized future. Perhaps also in the future which might be as well as the actual future will be." (p. 73, abridged.)

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And with this Whitehead has plunged into metaphysics, and we who remain on the dry land of scientific common sense (dry in more senses than one) are forced sorrowfully to part company with him.

III. *After the Fall: Metaphysical*

The mischief has now been done, and induction is the thing that has done it. The doctrine of universal connectedness and that of organic time as a justification of induction have led him straight into metaphysics. Yet this was not his only problem, and to a superficial view, not even a main issue in his thought. In the passage which I quoted (*ante,* p. 190) there were two other problems besides induction which pushed him into metaphysics: The Interrelatedness of the Various Types of Abstractions, and a Rationalization of his Epistemological Realism. Perhaps both of these should have been discussed along with induction in the preceding section. My only excuse for postponing them lies in the fact that they belong properly not to the transition period, but to the final stage. In his earlier pre-metaphysical phase Whitehead would not so much as sniff at them. When he starts taking them seriously we have a clear sign that he has broken his vows of knightly allegiance to his Dame Science, and has begun to flirt with the Lady of the Great Names.

A. *Types of Abstraction and their Interrelation.*

> "I hold that philosophy is the critic of abstractions. Its function is the double one, first of harmonising them by assigning to them their right relative status as abstractions, and secondly of completing them by direct comparison with more concrete intuitions of the universe, and thereby promoting the formation of more complete schemes of thought." (*Science and the Modern World,* p. 126.)

How far he has travelled from his original position must be obvious. He started not as a critic of abstractions in

general. He had only one set of abstractions in which he was interested, and that only in one definite way. He had before him the abstractions embodied in science. His problem was to determine the rightness of those abstractions in themselves: Were they legitimate refinements of the experience for which they stood? Now science is but a small part of the picture. He has before him the whole field of human experience, and the different techniques in which it finds expression: Besides the physical sciences we have biology and the social sciences, and over and above them we have poetry and religion. The field has widened. "Concrete experience" is no longer limited to the spatio-temporal connections of events: It includes such things as,

ORGANISMS, "Concrete enduring entities" where "the plan of the *whole* influences the very characters of the various subordinate organisms which enter into it" (*Science,* p. 115);

VALUES, which arise "from the cumulation, in some sense, of the brooding presence of the whole onto its various parts" (*Science,* p. 127);

and RELIGION, which "is the vision of something which stands beyond, behind, and within the passing flux of immediate things." (*Science,* p. 275.)

The moment Whitehead leaves science and deals with the abstractions involved in biology, poetry or religion with the same realistic method, trying to trace them back to concrete experience, he has immediately enmeshed himself in metaphysics. It was inescapable. It could have been avoided in one of two ways: Either by staying forever in his chicken-coop of science, or by adopting the method of Santayana, and treating all matters of value on the same basis as secondary qualities, i. e., as things to which the concept of truth cannot be significantly applied.

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I fail to see that the second of those alternatives is open to a consistent philosophy. If you are objective in your view of scientific objects, you must maintain the same attitude when you come to other types of experience. If electrons are facts, so are organisms, value, and God. Objectivist once, objectivist always.

That position, right or wrong, is the one that Whitehead now takes. If we have understood what it involves, we may go back to his definition of philosophy as the "critic of abstractions." It has a double task, we are told:

1. To determine the relative status of the different systems in an economy of abstractions, i. e., as refinements of a common fund of undifferentiated experience. They are further to be known in their own specific abstractive character. The realm of each must be outlined, so that there shall be no danger of confusing art for science, and religion for common sense.

2. To ascertain their adequacy by recomparing them with the concrete experience which they are supposed to represent. What has already been done for science (the relating of scientific abstractions to the data of native experience) must now be done for the other interpretations of experience. So far, Whitehead has only done this in a summary fashion for religion. His treatment here has the same merits and demerits as his philosophy of science: His work has been marked by brilliance, insight, and sureness of touch, but has not been exhaustive enough to give a complete philosophy of religion. His real work as "critic of abstractions" in science, religion, and elsewhere, has so far consisted in laying down a program and a method of approach, based upon a metaphysics, which both presents the presuppositions of the criticism and marks out the best way of attack.

B. *Epistemology and its Metaphysics.* Epistemology

is an old pet of Whitehead's. He started on it with the first chapter of his first book. Of course, he was not so ambitious then and contented himself with a realism that was merely provisional. Yet he decided upon his epistemology from the very beginning, and retained it unchanged to the last. The departure from it has been not in altering it, but in basing it upon a metaphysical foundation. His theory was extremely simple:

> "Natural knowledge is a knowledge from within nature, and is an awareness of the natural relations of one element in nature (namely, the percipient event) to the rest of nature." (*Principles of Natural Knowledge,* p. 13.)

Now an over-scrupulous reader might have asked him this question: If knowledge is a relation of one event to other events, how is it that this particular event (mind) has such a strange relation to the rest of nature, a relation which seems to be unique to itself and not to be reduplicated by any of its brother-events in nature? There are two ways in which this could have been answered: One would be to turn savagely upon the questioner, laugh him to scorn, call him names, metaphysical, behind the times, etc. Such an answer might silence the questioner but would not silence the question. The other way would be to give some explanation of this curious fact of knowledge in terms which would be applicable to nature as a whole, and would remove the status of mind from the list of natural scandals. Whitehead has adopted the latter alternative. Professing agnosticism at the time, he has since given an explanation in terms of a monadological view of nature, which is at least as satisfactory an answer as any that has been offered by traditional philosophy.

It can be summarized in the word *prehension.* Perception is but a form of this universal fact of prehension, which is the mirroring of the whole of nature in each

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particular event. The universal connectedness of things shows that prehension is not limited to man: All events are to some extent influenced by the whole of nature within which they occur. In Bacon's words, "all bodies whatsoever though they have no sense, yet they have perception (prehension), and whether the body be alterant or altered, evermore a perception precedeth operation." (*Science,* p. 100.) So human perception is "the prehension into unity of the patterned aspects of the universe. Its knowledge of itself arises from its own relevance to the things of which it prehends the aspects." (*Science,* p. 213-4.) This explains perception. It also explains much more: the internality of relation, and the continuity of events, where each is involved in all, if not as an "active" at least as a "passive" event.. By the concept of "relevance," which establishes different degrees of prehension, Whitehead is now able to justify the fact of "significance" and its correlative notion of the contrast of discerned and discernible.

C. *A Sketch of Whitehead's Metaphysics.* Whitehead's metaphysics is not the main object of this paper. It enters in only in so far as it represents the inevitable outcome of a development already implicit in his philosophy of science. Yet for the sake of completeness, I will give a brief account of the final metaphysical position. It will help to gather up all the loose threads which so far we have been following separately. Besides a clear statement of a metaphysics that has since achieved notoriety will render a service to all philosophy. It will at least show it up in its true colors, and perhaps correct some of the impressions that have resulted from the gossip broadcasted about by Kallen & Co.

Whitehead's metaphysics begins with the two great commonplaces of philosophy: the flowing stream of Hera-

clitus, and the frozen world of the Eleatics. Both are equally important, both equally impossible, and both equally obvious to the naive mind. On the one hand, universal change, perpetual flow, endless shift. This is the passage of nature. On the other, permanence, recurrence, fixity. This is the order of nature. In this simple contrast lies the basis of Whitehead's metaphysics.

The passage of nature does not occupy him long. It is there before you; open your eyes and you will see it. It needs no explanation beyond the assertion that it so happens. But the order of nature needs a little more attention. We can describe it (or explain it, for explanation, after all, is nothing else but description) in terms of the ingression of the eternal objects into actual occasions. These are big words, but be not frightened. A strange term does not mean a patent for the invention of a new entity, but only a copyright for a novel description of a familiar thing. Whitehead's bite is really much more harmless than his bark. All he means to tell you is to call your attention to the permanent elements in the world. These are the eternal objects or ideal entities. These entities have no existence apart from the actual world; they are not stored away, along with Plato's Ideas and Santayana's Essences, in some metaphysical storehouse, from which they are released a few at a time and allowed to return to the world of events. Yet that they do have some kind of being apart from any existent thing, is also true. World or no world, triangles would still keep their properties, and maroon its color. They would exist, but only as possibilities.

Ingression merely means the actualization of possibilities. With a little more imagination than we can expect from the common run of philosophers, we could ask them to imagine the world of actuality as a great castle of

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crystal, against which innumerable throngs of eternal objects flatten their noses, looking on greedily at what is taking place within, and impatiently waiting for admittance. Whitehead is more considerate. He knows how much (or how little) he can expect from the hard-headedness of professional philosophers, so he does not ask them to perform such feats of the imagination. He only states: That there is an actual, passing world; that in the universal flow of this passage certain elements persist unchanged, and can therefore be recognized by us; that these unchangeable objects cannot depend for their being on an ever-shifting world, and must therefore have some sort of independent subsistence; that the being of these eternal objects is not a ghost-like imitation of actuality, but consists in mere possibility that in this realm of possibilities essential characters and mutual relations are maintained, that circles keep their roundness, atoms their atomicity, and burnt sienna its brownness.

I have been watching you all along. Your wrinkling brow, your impatient eye-lids, your half-caught breath, have all been signs of a gathering storm, which I knew would break out as soon as I stopped. That is why I made my last sentence so long: I wanted you to hear it all before you speak. And now I hear your objections. The eternal objects scandalize you. But why? Let us not fight over words. Surely the permanences of things are obvious; you would not deny them. You are full of sarcasm for the realm of possibilities. But why? Are they not real in their own way, real and incontrovertible? It is possible that you will sneeze to-morrow at 10:00 A. M. What is there objectionable about that? You blurt out that it is meaningless, nonsensical, mere sound, etc. Not so fast, my friend. I think that possibilities make very good sense. Every time you predict the weather, make

plans for your vacation, and remark pleasantly to your wife, that "the worst is yet to come," you are moving in the realm of possibilities. Try to define a knife, and do it without referring to "its associated hierarchy of possibilities," its possible uses, history, price, etc. Or if you want to be scientific, what is your hypothesis, but such a possibility, which if successful, you shall witness in its ingression into actuality?

Now if we are sure that we understand each other, we proceed. Let us describe a little further the ingression of the eternal objects into events. This is obviously done through limitation. The only way you can get definite actuality out of indefinite possibility is by cutting down your alternatives. The minimum of limitation of the eternal objects is that of space and time; the least they can do is to keep to a certain place, and move according to a certain progress of time. But this will mean more than date and place. It will mean that they shall have to conform to what is actual and settled before their own arrival on the stage; they cannot create their own parts as they would like to have them, but must keep to their lines and fit into the plot. This is destiny, fate, durée réelle—call it what you will. It is the irrevocableness of the past, upon which the present must always build.

Yet the locus of spatio-temporal determination is not all that is necessary for the final form of actuality. Within the limitation that the antecedent ground imposes on the derivative consequent, there is still an indefinite number of possibilities out of which shall come the unpredictable freshness of the present. What is the final determining principle that makes actuality what it is? Explanation here comes to an end. There is no reason that we can give for the particular form that actuality assumes from

moment to moment. This is the ultimate irrationality. It is God.

Eternal objects, spatio-temporal limitation, causal push, the drag of the past, God—all these form the order of nature, and order in its intersection with passage issues in actuality. Every actual occasion mirrors in itself the whole world of past events and the whole world of ideal possibilities. It is a prehension of the whole into a part, and such a prehension means value. Everything that is actual has value, and its value is due to the purpose of God, who so envisages the realm of possibilities and the world of settled fact as to focus them upon each actual occasion for the creation of something new.

Here we may call a halt. All the essential elements of Whitehead's metaphysics are now before us. I started out to give an impartial and objective statement, but much to my surprise found myself taking an apologetic attitude. I did not do this because I agree with Whitehead. All I cared for was to make him understood. I am not inclined to defend the whole of his metaphysical system. I believe that his fundamental description of the passage and the order of nature is only a matter of terminology, and can be accepted by any fairminded man who understands what Whitehead is talking about. The further developments of prehension, value, and God are debatable. Even there, if one understands Whitehead's definition of his terms and uses them strictly in the specified sense, much mischief will be avoided. The greatest danger is not from disagreeing with Whitehead, but from misunderstanding him.

Conclusion: The Unescapableness of Metaphysics

Every tale has a moral, though the real moral is not always the most evident one. Take the Adam story, for instance. A superficial reader may think that its significance lies in the dire results of disobedience. Yet there is

something else, less obvious, but more true to the spirit of the account. It is the inevitableness of the fall. Given a curious Adam and a forbidden fruit, you know that the fall is inevitable. If you are a supralapsarian (as you cannot help being, if you have thought the matter through), you say that God knew it too, and foreordained that things should happen that way. Likewise the most obvious moral of Whitehead's story is that a philosopher of science, who is a philosopher as well as a scientist, must watch his step, or else he will land in metaphysical hysterics. This can be explained, elaborated on, and enforced in many ways, by meditating upon the greatness of the fall, and calling upon the younger generation of philosophers to beware of the deadly attraction of metaphysics. But for me this is not the only moral of the story, nor indeed the truest one. The most striking feature in the transition of Whitehead's thought from the *Principles* to *Science* is the fateful unavoidableness of the development. The moral: A philosophy of science that is more than a philosophy of science, if it is still a philosophy, must be a metaphysics. Let me explain.

What is a metaphysics? A metaphysics is a rationalized account of the world that says what the world is if our various types of experience are to become intelligible and harmoniously interrelated. A metaphysics takes the concepts of the various techniques of experience; it analyzes the abstractions of science, art, and religion, and determines whether they are legitimate and adequate within their own field; then emerging out of the split-up worlds that each type of experience presents, it takes a broader view, and freed now from the short-sightedness that is the price of immediacy, it sees science, art, and religion not as rivals but as correlatives; basing itself upon the axiom that, what make sense severally must make sense

collectively, it tries to effect a universal synthesis. Now while synthesis must be built with the most vigorous observance of the law of structural economy, it must still be a synthesis, and a certain amount of interpretation will be found absolutely necessary, if the various types of experience are to find their place as parts of one organic whole. The end of all philosophy is the synthesis; the problem is, how to achieve it with the least amount of rationalization. "Metaphysics" has been used in the recent past in a derogatory sense to refer to the reckless use of generalizations, and conjectural simplifications of the universe unsupported by fact. My contention is that some amount of rationalization will be always necessary: you cannot banish it from philosophy, any more than you can eliminate hypothesis from science. To condemn any rationalization as metaphysical and therefore unworthy of the attention of sensible people, only shows an unfortunate prejudice, which itself is based on the generalization that all rationalizations are bad. Acknowledging the term metaphysics, and giving it the wider meaning of a rational interpretation of the variously experienced universe, I would demand that every metaphysics be judged on its own merits, according to the excellence of the synthesis which it effects, and the justification of the rationalizations which it involves.

This definition of metaphysics must have made my previous statement clear. When Whitehead left the philosophy of science for a wider view of the universe, he could only avoid becoming metaphysical at the price of being synthetic. He cast in his lot with metaphysics, and achieved a certain measure of intellectual completeness and aesthetic elegance by basing his whole philosophical structure on a single generalization: Prehension. We saw how he arrived at this prehension not by a feat of imagi-

nation or a leap of faith, but by a slow, logical process in the attempt to complete his philosophy of science. Prehension seemed the best way of answering the riddle of induction.

It may be that this is too bold a solution, that it is not sufficiently grounded upon facts, and by its very nature incapable of direct verification. If that is the case, it is hard to see how it can be accepted as the solid rock on which to rear an unshakable philosophy. On this point I cannot offer a definite answer. It is not the ambition of this paper to settle any metaphysical problems. My aim was the much humbler one of showing how a logical, clear, and exact thinker, starting out from the philosophy of science, ended in metaphysics, and that, not because he loved fact less but loved synthesis more. How much this proves I cannot say. If I had not been properly indoctrinated in the dangers of generalization, and taught that induction itself is one of these unfounded generalizations which are to be avoided like the pest, I would have drawn the obvious conclusion:

> Whitehead is a philosopher of science;
> He could not complete his philosophy of science without falling into metaphysics;
> Metaphysics was inevitable for Whitehead;
> Metaphysics is inevitable for every philosophy of science that is complete.

If my reader is less inhibited, he can draw the conclusion for himself.

GREGORY VLASTOS.

CHICAGO.

PLATO'S CONCEPTION OF THE FUTURE AS OPPOSED TO SPENGLER'S

MY aim in this paper is to determine if Spengler's dogmatic assertion that the Greeks had no sense of the future is valid in so far as the evidence of Plato's dialogues is concerned. *First,* I shall briefly state Spengler's own conception. *Next,* I shall discuss more fully and in detail Plato's conception of the future. *Finally,* it will be possible to compare the relevant items of the two conceptions, and thus modify, if possible, the still undisputed assertion of Spengler.

Spengler's monumental work *The Decline of the West* offers a tragic yet fascinating metaphysical theory of the future. In it the world as history passes in panoramic and plastic pictures before us. Historical events are treated as things becoming, as actualities, and not as things become. In its general form the future is a zoological perspective of consciousness. "To man in the waking state his proper life, progressive and constantly self-fulfilling, is presented through the element of becoming in his consciousness—this fact we call "the Present" —it possesses that mysterious property of *Direction* which in all higher languages men have sought to impound and—vainly—to rationalize by means of the enigmatic time." "The soul as it is felt is designated as the possible and the world as the actual, then life is the form in which the actualizing of the possible is accomplished. With respect

to the property of *Direction,* the possible is called the Future, and the actualized the Past."[1] The nature of future cannot be apprehended by the cognitive forms of "pure reason." It is inwardly lived, experienced and intuited. It is radically chronological for Spengler's mind. It is the spontaneous yet necessary aspect of what we call time. It is the metaphysical essence of history. Historical acts *per se* are not exhausted by their appearance. They one and all point to something by appearing. Time, then, is for Spengler essentially an inner direction, distance, farness. Its implied futurity is identical with the direction, impulse, will, movement-quality of living and organic beings. As such it is indivisible, irreversible, once uniquely occuring, not determinable and repetitive either mechanically or causally. Future is *Destiny* that we live and are. Our span of life and that of cultures, is a fated-coursed time, the infinitessimal growth of the soul passing through adolescence, maturity and decay to its self-fulfilment. Spengler conceives future to be an organic, dynamic experience. It possesses a vertical tendency, consists of a Faustian striving upward until the process of its inward form has attained its organic completion. The unforeseeability of the future, that which men longingly and anxiously live forward to, drives them to an end they really do not know, but which they live out. It is thus an organic necessity of life itself. There is, however, no conscious teleology, plan, or aim in the future as Spengler knows it.

Spengler deduces from the above major characteristics of his conception of the future the allegation that the typical Greek was by constitution incapable of conceiving of a future, because he possessed no intimate sense of time. The specific points in Spengler's denial are: 1. The

[1] *The Decline of the West,* p. 54.

Greeks possessed no historical memory. They had no impression that their life was a part in a wider life-course. They were devoid of any world-feeling (p. 8). They had no pious preservation of the past, and they conceived of no work for the benefit of future generations (p. 132). 2. They transmuted all their experiences into a timeless and immutable background. They wrote down no recollections that would serve to fix a phase of experience for their inner eye (p. 14). They were not deeply personal; but immediately transmuted their experience into what was typical and universal. 3. They localized time to a definite place and instant, and possessed no extension of time into space. No time-reckoning entered intimately into their daily life (p.9). 4. They did not conceive the world as continuous but as complete. 5. The Greek soul was Apollinian, while that of the modern Western World is Faustian in quality. The former is essentially rest and rational clarity, while the latter is unrest and longing. Rest presupposes the feeling of being and of existence; unrest has the consciousness of becoming. Clarity deals with what is limited; aspiration with the unlimited and ineffable.

Plato had not only a general notion of the future; but he assigned diverse meanings to the concept. We shall discuss the various meanings of the Platonic concept of future in the following order: 1. as an unspecified mode of time, 2. as a mode of technical improvements, 3. as a mode of organic evolution, 4. as a mode of physical chronology, 5. as a mode of physical growth, 6. as a mode of moral evolution, 7. as a mode of intellectual development, and 8. as a mode of political progress.

I. Of all the organic beings in the realm of nature it is probably man alone who acquires in the course of his development a sense of the future. The sense of time is at

first largely confined to a confused consciousness of the present; but later as experiences accumulate, a retrospective and a prospective attitude develop. The Greeks were no exception to this general human characteristic. Plato shared with them this unspecified and common trait. Future was for him, as it was for them, a direct outgrowth of of a present situation, either intended or suggested in the course of mental preoccupation, or a mode of temporal distance in terms of astronomical time.[2]

II. Though a thoroughgoing idealist in philosophical speculation, Plato set a high value upon technical discoveries. In the third book of the *Laws*[3] we see him recognize the great merits of the inventions of the simplest tools and arts. Very instructive is what Plato accuses Homer of in the tenth book of the *Republic*. No city has anything to say about him. No war has been successfully fought under his leadership, nor has he made any technical inventions like Thales of Miletus, or the Scythian Anacharsis, the inventor of the bellows and the two-sided anchor.[4] In Plato's Academy ample use was made of planetarias in the study of astronomy. In the *Philebus* Plato makes the pregnant declaration that the progressive and productive technical advance of the various arts depends upon a corresponding advance of mathematics. Without it the arts will have little chance of survival. Plato is keenly aware that the cause of the unproductive status of contemporary music must be ascribed to the absence of a scientific study of the art. He feels the same holds true of contemporary medicine, agriculture, navigation and military tactics. He knows that the technical advance in ship-building and

[2] *Laches* 181, 201; *Charmides* 157, 169; *Protagoras* 314, 361; *Gorgias* 447; *Rep.* 363, 461; *Laws* 656, 753-4, 826, etc.

[3] *Laws* 677-78.

[4] *Rep.* 599-600.

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architecture is due to the growth of mathematical knowledge.[5]

III. It is in the *Timaeus* that Plato furnishes a general outline of the future as a mode of biological evolution. The thesis is suggested there that the investigation of the physical universe has the sole aim of better understanding man's nature and position in the scheme of things.[6] Man is for the author of the dialogue the end of organic and inorganic evolution. All animals, plants and inorganic structures are for man's sake, and at his service. Prior to the evolution of organic beings were the planets, which Plato ranks as the most superior class of living beings. The others he differentiates into those of the air, those in the water, and those on *terra firma*.[7] Later in the dialogue, the author attempts to explain the above threefold division of organic beings in terms of their differences of organization. He sets forth the idea that the earth first formed the males. Those of the originally created males, which proved themselves cowardly and unjust in the first generation, were transformed into women. Moreover, not only the women but the animals evolved from the originally created males. The animals came into being as a result of an increasing degeneration among men.[8] If we abstract the mythical character of the story, the following important notions emerge: that the diverse embodiments of the animal type correspond to diverse degrees of ensoulment; that in man we have the superior developmental form of animal life; and that between him and the animals there is no inseparable chasm. Plato made the significant observation that one could not correctly understand the nails of man apart from the nails of the animals.[9] Stripped

[5] *Phileb.* 55-56, *Symp.* 209.
[6] *Tim.* 27.
[7] *Tim.* 39-40.
[8] *Tim.* 41-2, 90-2.
[9] *Tim.* 76.

of its mythical garb, Plato's account of organic evolution does no longer appear as a descending evolution by degeneration; but as an ascending evolution. The intimation that there is also a mental kinship between animals and men, is found in the *Statesman* 263 D and in the *Republic* 376.

We have seen that Plato shared with his fellow-Greeks the notion that future is a genuine experiential aspect of the world of flux. In true Greek fashion, he also partook with his fellow men the ineradicable urge, human need, to seek as a speculative thinker, an escape from the world of time. There must be finalities which enable the restless soul to transcend the relativity of the temporal world. By patient intellectual and moral toil Plato found his answer and solution in the world of *Ideas*. The ideas are eternal, immutable, constituents of a higher world. The temporal world participates in part in this world. Having by patient endeavors come to see the ideas in their abiding essence and relations, Plato, like the ideal philosopher he was, turned back to the world of flux with the sole intent to implant in a few radical ways the ideals of the eternal world into its textures. In other words, he conceived future to consist in a perpetual application of the ideal to the temporal. His supreme passion was to traditionalize, idealize and rationalize the eternal ideas in the actual world of time. Therefore, in delineating Plato's conception of future as a mode of physical chronology, of physical growth, of moral advance, of intellectual progress and of political improvement, we must above all be mindful that the peculiarly Platonic notion consists in the bold application and adaptation of the ideal to the world of actualities, in order to ensure its future continuance there.

IV. It is in the *Timaeus* that Plato sets forth his conception of the future as a mode of physical chronology.

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The *Idea of Good* is there put forward as the goal of divine activity, as the typical idea to which divine intelligence looked and by which it was guided in the creation of the cosmos. The realm of physical nature was not a cosmos until it had been remodelled and refashioned by the genius of the divine artist. Both the *Idea of Good* and God, as the creator, are spoken of as constituting together the creative principle.[10]

There is no divine creation possible except by imitation of the ideal. God's creative activity consists essentially in working out the proper combinations of ideas with the phenomena of the world of becoming, in forming the universe according to ideas. The creator brought the pre-existent chaotic matter into order by introducing into it form and number. He introduced the principle of mind into the disorderly and irregular physical world. Thus it became a rational, visible whole, comprehending within it all other parts (wholes) of kindred nature. It received a fair and perfect body.[11]

God applied the principle of the *Good* and the *Mean* in creating the physical universe. He compounded the cosmos out of fire, earth, water and air, which in their pre-existent condition were in a chaotic state of motion. No part of the pre-existent world was left unaffected by God's creative and ordering activity. In creating the body of the universe God placed its elements in proportion to one another as far as it was possible. Being a solid, the world comprehends within itself all other three-dimensional bodies. God placed the two means, water and air, between fire and earth. Next, the world was given a suitable figure, that of a globe, consisting of equal distances from the centre to the circumference. It further was endowed with an appropriate motion, moving in a uniform circular

[10] *Tim.* 29.
[11] *Tim.* 28-30.

manner on the same spot. The universe being created thus, became indissolubly one, perfect, immortal, self-sufficient, self-maintaining, and above all intelligible.[12]

At the centre of the sensible and tangible cosmos was the soul, diffusing and enveloping its body. In origin the soul is among the first of bodies, the chief author of their changes. Hence mind and its ideas are prior to the corporeal. As a self-moving principle, the soul or mind sets everything in motion. The true form of its movement is circular. It controls the world according to harmonic and mathematical proportions. Plato finds that there are two conflicting souls (motions) in the world, the one good, the other bad. The former moves according to number and law, its motion is regular and uniform; whereas the latter moves irregularly, lawlessly and without proportion.[13] It was because of the creation of the soul that the physical universe began its perpetual rational life throughout all time.

The cosmos was ideally conceived by the mind of its creator. It was ideally moving in accordance with mathematical and harmonic proportions. In order to realize the ideal cosmos in the chaotic flux of the world of physical space, God created a moving image of it, time. But in order to create time in conformity to the ideal, he fashioned it simultaneously with the planets and stars. When God introduced order into the pre-existing world of physical space, he endowed time with a motion according to number. The ideal or eternal does not move. It rests in absolute unity. Hence time must possess a motion which proceeds according to mathematical and harmonic proportions. The eternal has no motion; but what is created has motion. Time thus constitutes for Plato the connecting link between the ideal world and the world of becoming.

[12] *Tim.* 31-34.
[13] *Laws* 892-98.

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It is the medium by which order enters into the world of space.[14]

Next, Plato attempts to show how the ideal elements of mathematical extension mediate between the ideal world and the world of chaotic physical space. Plato believes that the world of physical space can attain in an extraordinary way a portion of the intelligible. It can be made rational by the imposition of mathematics. Physical space can, however, never fully attain rationality, because what is sense-perceivable of matter cannot be adequately accounted for by mind.[15] Chaotic matter or physical space consists of powers which are neither similar nor equally balanced. Its movement is disorderly. There is neither measure nor reason in it, only blind necessity and chance, mere mechanical occurrence as a result of universal pressure. Its incomplete bodies or elements group themselves according to their likeness. It was, however, only by the intervention of the divine creator that the incomplete bodies of physical space were fashioned by forms and number.[16]

The primitive elements, being solids, consist of planes or surfaces. All surfaces in turn are composed of triangles, and all triangles arise out of two different equilateral triangles, the isosceles and the scalene. The triangles are the original elements of the four kinds of bodies. The most beautiful and best triangle for the formation of the elements is the rectangular triangle. Out of six such triangles arises an equilateral triangle, and out of four isosceles triangles arises a square. Out of the square is formed the cube, the form of the earth. The tetrahedron is the basic form of fire, the octahedron that of air, and the icosahedron that of water. Three of the

[14] *Tim.* 37-39.
[15] *Tim.* 48-52.
[16] *Tim.* 52-53.

four elements are thus generated out of a combination of equilateral triangles, and one out of the isosceles triangle.[17] Plato furnishes the numbers and formulas of combination of each of the elements. The dodecahedron is not classed by Plato as one of the ultimate elements. It is of all regular solids that which nearest approaches the sphere, and that which could be most readily hypothecated as imaging perfectly the plan of the universe. It is evident that the ground underlying the geometrical derivation of the physical bodies is mathematical extension. The elementary geometrical figures, which are at the basis of the actual physical bodies, are the ideal elements which the divine creator introduced into the unformed bodies of the world of physical space.[18]

V. Plato possessed a definite ideal of a perfectly formed and healthy human body, which he desired to realize within the confines of the actual world. He was fully aware that the fortunes of human life were subject to the temporal law of succession, of degeneration and death. The problem, therefore, was to demonstrate how the ideal physical body could be preserved through the indefinite succession of future generations. In order to realize his ideal, Plato advocates a state-controlled heredity of family life, to make possible the production of physically perfect children. During the period of procreation men must abstain from dissipation by intoxication, as there is every likelihood that their offspring will be affected by it. The mode of living is prescribed for the prospective parents during the year before the children are born. The pre-natal care of both the infants and the prospective mothers is considered. Exercises are to be imposed upon the unborn infants by having their mothers walk about. The prospective mothers are carefully tended and kept

[17] *Tim.* 53-54.

[18] *Tim.* 33, 40, 54-55.

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from violent pleasures and pains. In order to carry out the afore-mentioned legislation with the least possible compulsion, it behooves the leaders of the state to elevate the regulated reproductive urge to a principle of piety. The people must be enjoined that it is their pious duty "to rear children which will hand on the torch of life from generation to generation." They are taught that they ought to cling to a biological mode of immortality in order that their posterity may serve God in their stead. Marriage shall be made a sacrament, and holy festivals shall be instituted to the end that the ideal of physical perfection may find its concrete embodiment.[19]

After the children have been well-born, it devolves upon the state to see to it that they have a good and sane physical education. Though the body starts in its post-natal phase with a sound constitution, it has need of continued physical training, if its health and perfection are to be ensured. Children should not be carried about after they have reached the age of three. Sports and exercises are required by the child of three to six years of age. Special meeting-places must be provided for them. When six years of age, the boys learn the use of bow, javelin and sling; the girls, the use of arms. The gymnastic exercises of children comprise two branches: dancing and wrestling. The children are taught to use both hands skilfully. The trainers are careful that both sides of the body are developed symmetrically. Plato does not allow any innovations to be made by either the children or the trainers for he desires the traditionalization of his ideal scheme of physical education.[20]

Certain general principles of bodily care and diet shall be observed. A simple diet will not incline youth to have much medical care. All refinements and sweets must be

[19] *Rep.* 423, 459-61, 468; *Laws* 773-792.
[20] *Laws* 778, 793-800; *Rep.* 466-7.

excluded. It is only by a regulated system of gymnastic and diet that new diseases are held in check. Illnesses and hospitals must not be encouraged by a life of indolence and luxury. Only those of sound bodily constitution and clean habits deserve medical attention. Totally diseased bodies are not worthy of cure. After the ideal physical education has become an established tradition, the bodily health and excellence of the citizens will go on of itself, and the race will improve more and more.[21]

During the early years of an individual's life great care should be expended on bodily improvement. In later youth and mature life, excessive bodily care is inimical for the good of the individual and the state. Mere gymnastic exercises are not sufficient for growth in physical perfection and health. Hence bodily training must be supplemented at all stages by an equally strenuous training in virtue and intellectual insight.[22]

VI. Education even in the earliest stage of life is not exclusively physical. Certain special tendencies which begin to show themselves at this time require moral direction. Young children seek satisfaction and avoid dissatisfaction. This general tendency expresses itself in them in a desire for simple sense-pleasures and avoidance of pain. They express these wants during the first three years by crying, so as to attract the assistance of parents and nurses. If this tendency to cry whenever the child is afraid, or wants anything, remains unchecked, it may lay the physical foundation for dispositions, which if further developed, will grow into vicious habits. The task of parents during this period is to prevent the formation of bad habits. Their endeavors must be directed to develop in the children a general disposition towards moderation in matters of pleasure and pain experiences, and foster the

[21] *Rep.* 403-405, 407-410, 432; *Statesman* 306-7.
[22] *Rep.* 403-4, 407, 411.

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positive habits of self-control, manliness and serene temper.[23]

The next stage starts with the third or fourth year. At first, the period is still largely confined to parental education; but later it is in the hands of professional teachers. Education is introduced through the medium of music and literature. The literary studies comprise chiefly poetry and prose. Musical training consists mainly in simple accompaniments upon the lyre, used in connection with the songs and dances taught to the children. Plato recognizes the tremendous influence of literature and music upon the habits and dispositions of the growing child, as these condition by means of imagery their future character. Tales about wars among the gods, lamentations of death, laughing gods and heroes, poetry that implies that wicked men are happy and good men miserable, are not permitted in the program of moral education. The future guardians shall imitate and dramatize those virtues which are in accord with their ideal nature. They shall physically and mentally be taught to act unconsciously according to the principle of virtue. Therefore, only such art and literature is encouraged as is calculated, in the opinion of the best judges, to develop and strengthen the natural dispositions of a balanced character. No innovations are allowed unless these conform to the principle of virtue. Plato intimates that this grounding of ethical dispositions and habits prepares the mind for the gradual development of reason. A beneficient authority is exerted over children and youth, and freedom refused them, until the time when, by cultivation of virtue, they have established in their being a ruler. Now they may go their ways; for now their moral growth will go on of itself.[24]

VII. Hereafter, we shall discuss Plato's conception

[23] *Laws* 792; *Rep.* 605.

[24] *Rep.* 377-8, 383, 387-92, 394, 401-2, 590-1; *Laws* 376-7, 656.

of the future as a mode of moral and intellectual progress together, as they never really stand apart in Plato's mind. Starting a little later than physical education, comes intellectual education proper. The major aim is to liberate the intellect from a too narrow preoccupation with the sense-perceivable world by means of special studies, to direct its powers towards the apprehension of the intelligible world. This educational stage falls into two parts: the preparatory portion is a matter, partly, of a moral, and partly, of a specifically intellectual training. On the moral side, it is necessary to liberate the instincts and habits from their excessive preoccupation with the sense-perceivable world. Due to the fact that this liberation is only slightly effected by means of the moral influences, a deliberate policy of selection is applied to and imposed upon the students. They are being tested by specifically determined character tests. Only those who show marked proficiency in moral improvements, in control of instincts and habits by ideas which were socially implanted in them, and who show a sound memory, love of study and intellectual ability, are permitted to proceed with the intellectual training.[25] The intellectual training in this period of preparation consists in the study of arithmetic, plane and solid geometry, astronomy and harmonics. The subjects are studied simultaneously, though harmonics is not commenced until the student has made progress in astronomy. The study of astronomy presupposes solid geometry; solid geometry presupposes plane geometry, and this in turn arithmetic. Up to the age of 20 these subjects are studied separately; after that in their interrelations and philosophical significance.[26]

The aim of moral education is to use these disciplines as a means for effecting a certain revolution in the intel-

[25] *Rep.* 535; *Laws* 647-9.

[26] *Rep.* 521, 527-31, 537; *Laws* 818.

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lectual outlook of the students. In the ideal system of education advocated by Plato the radical element consists in the stress laid on theory for its own sake, and the consequent long postponement of the solution of practical problems. The practical difficulties are employed as intellectual stimuli, which in turn awaken further intellectual interest, until the scientific habit of mind is formed, and the student feels at home in the world of laws and relations. This can only be achieved by a long diligent training of intelligence. The period terminates in a sustained investigation of the interrelations of the various sciences. It is followed by a stage of further intellectual toil in which the students study the basic principles of social and religious life. It serves as a final step, preceding the entrance upon the stage of dialectics.[27]

This last stage of intellectual training is undertaken at the age of 50. Only those take it, whose moral and intellectual equipment is of the highest, whose practical experience and strenuous study of religious and moral principles have fitted them to take the final step. During the whole period of preparation (from childhood to the age of 50) the whole nature of the future guardians is made over, and their character is based upon the eternal ideas. By continuous habituation and practice, the ideas of justice, courage, temperance wisdom, have become constituents of their very nature. Intellectually the prospective guardians have become accustomed to think of the interrelations of the sciences as pathways to reality. Their nature, therefore, has become a fit habitation of the *Ideas*. Having thus developed a personality which truly reflects the *Ideas,* they come to apprehend the total meaning of reality, the principle that makes everything one and rational. The final step that leads to the vision of the ideas as forms of

[27] *Rep.* 498, 523, 531, 537.

the *Idea of Good,* entails severe intellectual toil. The various departmental sciences and the social and religious institutions constitute the point of departure in this philosophic quest. Having ascertained the presuppositions of every science, the full guardians convert them into so much material of investigation, seeking its ground in a higher and more rational principle. Thus they pass beyond the apprehension of ideas as mere empirical generalizations, and apprehend the ideas in their true essence, set free from the limitations of sense-experience. They apprehend the ideas as true universals.[28]

VIII. The fortunes of the ideal State within the world of time are indissolubly tied up with the production of the philosopher-king, whose physical, moral and intellectual growth we traced. The paramount question is now: What ought the actual political State do in order that the production of the ideal ruler may be assured? Traditionalize the ideal program of physical, moral and intellectual education. We also know what the ideals of the ideal ruler are. He is a thinker who has a clear grasp of the *Idea of Good,* possessing a universal knowledge. His gracious mind integrates all the excellences of his subjects, and regulates the whole life of the State with a view to the acquisition of virtue. He duly supervises the education of the rising generation of leaders, not permitting them to lead a secluded contemplative life, when their turn has come to take charge of the ship of State. It is thus that he and those whom he leaves as his trained successors, represent the embodiment of the future preservation of the State.[29] The philosophic statesman does not bother himself with the minute unforeseeable details of the traditionalization process of the ideal State. The endless reforms which now complicate political advancement and

[28] *Rep.* 518, 521, 527, 531, 533, 537, 540.

[29] *Rep.* 484-6, 489, 491, 499, 518-9; *Laws* 630, 739, 755, 770, 807.

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improvement realize themselves without the enactment of an unreasoned amount of legislation. But what is the least change the temporal State must undergo in order to pass into a more ideal form? There are two: philosophers must become kings, and kings, philosophers; women, children and possessions must become State-property. The ideal ruler will discover omissions in the realization of the ideal State as time goes on. Hence those who succeed him must have the opportunity of perfecting the design in actuality. If the absolute idea cannot be attained, the second best policy will do until possibilities present themselves for further approximation to the ideal. Here too, it holds, that if the ideal State has once made a start in the world of becoming, it will go on realizing itself in the indefinite future.[30]

Having traversed at length Plato's conception of the future, we now shall return to a critical consideration of Spengler's various assertions as to the inability of the Greeks to experience and conceive of time as either past or future.

1. Contrary to Spengler, Plato offers in the *Critias* (108-120) a lengthy account of the prehistoric and historic past of Greece. In the *Laws* he intentionally seeks to preserve the customs, practices and beliefs of primitive society. He desires that the popular faith and time-honored religious worship be maintained for the State (480,498). Then too, his writing of the Socratic dialogues is itself a refutation of Spengler's assertion that the Greeks possessed no pious preservation of the past; for Plato seeks here deliberately to conserve the ethical genius of his immortal master. In the *Menexenus* Plato has Socrates recount the funeral oration of Aspasia, in which she praises the noble departed for the goodness of their birth, nurture, education, the nobility of their deeds,

[30] *Laws* 745-6, 769, 770, 807; *Rep.* 415, 425-6, 464.

the country which maternally cared for them. She lauds the ancestors of the noble departed for their political virtues and patriotic deeds. In pleading for the continuity of virtue, she appeals to the living and future citizen (236-249). Plato conceived not only the *Menexenus,* but also the *Republic* and the *Laws* as an ideal work for the benefit of future generations. Like the French philosopher, A. Comte, Plato ordains that distinguished men shall after their death, be reverenced as gods (*Rep.* 540).

That the Greeks were devoid of any world-feeling is refuted by the evidence in Plato's *Laws.* Plato there demands that justice be done to the stranger whether he be accused rightly or falsely (879,938). He contends that xenelasia (banishment of stranger) is uncivilized. The citizens of the ideal State should show themselves worthy of being thought well of by the other races and the world at large. The public ambassadors that are sent to study the civilization and culture of foreign peoples, are enjoined to preserve the good reputation of their country by their conduct. They are definitely chosen for the sake of learning from those among whom they sojourn (950-1). It would indeed be strange for the Greeks not to have possessed a sense that their life was a part in a wider life-course; for they colonized and emigrated for material as well as intellectual benefits. Did not the Grecian emigrants when returning home give evidence of a wider mental horizon? Then too, the Greek deification of nature points likewise to the fact that they joyfully and tragically felt their kinship with a much wider life-course than just their immediate subjective existence.

2. That the Greeks readily transmuted all their experiences into a timeless and immutable background is certainly correct in the case of both Socrates and Plato. Theirs was the gift of objectifying, of extracting the

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typical and the universal in their experiences. But it does not follow as a corollary that they were not deeply personal. In all the Socratic dialogues we find ample evidence that whenever Socrates is not speaking philosophically, he is quite personal.

3. We have seen that time constitutes for Plato the connecting link between the ideal world and the world of becoming, the medium by which the creative principle of order enters into the world of physical space (*Tim.* 37-39). Hence Spengler's allegation that the Greeks possessed no extension of time into space is invalid as far as Plato is concerned. If time-reckoning had not intimately entered into their daily life, how would it have been possible for their thinkers to philosophize about time and the world of flux? It is rather because they took time seriously that they succeeded in taking a detached point of view of it, and passing beyond it, they extracted the sting of its pessimistic relativity.

4. Spengler's statement that the Greeks did not conceive of the world as continuous but as complete, is a half-truth as far as Plato's position is concerned. The world is for him ideally and conceptually a complete, systematic and unitary whole; but in its aspect of the world of time and of becoming, it is infinitely continuous.

5. Neither does the assertion that the Greek soul was exclusively Apollinian, i. e. essentially rest and rational clarity, as opposed to the Faustian, being unrest and longing, accord with the Platonic evidence. It is rather because Plato shared with his fellow men the intense consciousness of the relativity and unrest of the empirical world, that he yearned to rationalize and escape it. When he discusses in the *Laws* (892-898) the eternal conflict of the good and bad soul, the evidence shows that he was by no

means the placid idealist who with a gesture denies the reality of the temporal world. It is rather because time was awfully real, inspiring them with awe and fear, that the Greeks rose by the aid of concepts and philosophic ideas beyond it.

The Platonic conception of time possesses the very property of *Direction* which Spengler claims to be totally absent from the Greek mind. The ideal, being totally outside of time, had for Plato the value of a possible realization in the temporal world. The ideal, or the possible, was future as far as its realization is concerned. The life of the present was for him as it is for Spengler the actualizing of the possible or ideal, and the past was the record of the accomplished actualization of the ideal. Plato was firmly convinced that when the ideal has once taken root in a few places of the temporal world, it will realize itself more and more by the sheer force of its own momentum.

It is hardly in accord with the evidence that the Greeks possessed no pronounced sense of time. No one of the later thinkers was as strongly imbued with the reality of time, the flux of things, as Heracleitus of Ephesus. In comparison with the present-day thinkers like Bergson, Alexander, Whitehead and Einstein, the Greeks took time quite as seriously. The only difference is that the Greeks disposed of its problematic character more readily than recent thinkers and scientists. The Greek thinkers found it probably less difficult and amenable to solution. On the face of it, it is self-evident that Spengler's philosophy of time is in the main a modern extension of Heracleitean views in the mold of Nietzschean and Goethean romanticism and realism, buttressed by the advances of the biological and social sciences. Thus Spengler opposes one Greek conception of time to that of the Eleatic conception,

and then deduces without evidence that all the Greeks held to the Eleatic negation of time. It follows from the above considerations that Spengler's flat assertion of the Greeks not possessing any sense of time and of future is invalid in its major contentions, and that his own views are a direct modern extension of the Heracleitean, and in part, of the Platonic conception of time.

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NECESSITY IN THE PHILOSOPHY OF GIORDANO BRUNO

I

BRUNO'S rôle in the development of Copernican cosmology has often been described as that of propagandist. Bruno was a propagandist, but he was also a seer, who perceived more in Copernicanism proper—i. e., in the heliocentric theory—than did Copernicus himself. Indeed Copernicanism, partly because of Bruno's efforts, has come to bear a significance broader than that of any mathematical or physical doctrine. To-day Copernicanism suggests a point of view positing such fundamental premises as to merit the name of metaphysical scheme.

This expression 'Copernicanism' perhaps is inapt. Its essential meaning antedates Copernicus. Cusanus, Thomas à Kempis, Augustine, Democritus, are a few of the men who were imbued with its general spirit. Calvin, a contemporary of Copernicus, held much in common with the implications of an astronomical scheme he ridiculed.

We do not intend to suggest that these early thinkers believed in the heliocentric theory. Any of them, except Democritus, would have shuddered at it. The point is, Copernicanism in its broader sense means more than the heliocentric theory, and this broader sense can be traced through the thought of certain mediaeval philosophers.

A better name for our world-view would be 'telescopism' if astronomical associations of that word did not

lead a person still to mistake a particular form of the doctrine for its essence. 'Telescopism,' however, suggests the central tendency of looking away from human nature at distant things, enlarging them, bringing them near, while they, peering at man through the big end of the telescope, see him as diminutive and indistinct. The word telescope means 'to see afar.' If we emphasize the 'see' we have bared one element of the doctrine. Distant things of which man is aware are the entities to which he attributes fundamental importance. If we emphasize 'afar' we have isolated the other element. He beholds man from afar, thus barely distinguishing him. The thinker has stared long at some great being, calmly aloof, far above in its celestial throne, has unconsciously identified himself with that being, then looks back at his former place, and judging by his new standards sees it as small and futile.

If we want to behold this world-view in its pure essence and to free our minds of any bias towards its astronomical aspect, we might give it a less compromising name than 'telescopism.' The clumsy title 'doctrine of cosmological perspective' would be non-committal. It would link the under-current of thought neither with its grossly materialistic expression of pre-Socratic days, nor with its theological form of the middle ages, nor with its astronomical associations of to-day. We might impute the doctrine of cosmological perspective to Bertrand Russell or to Thomas à Kempis, though by so doing we can well imagine the shades of the latter to be seriously agitated. We must think, however, of cosmological as referring to any kind of universal order transcending man.

This matter of naming is unimportant. We have bothered with it in hope of getting an expression which would tersely reveal distinct wholeness in a great tendency of thought.

We might do this better by outlining the tenets themselves of our doctrine in terms not compromised by astronomical or other vesture. Four of these are essential. (1) Reality is infinite. We do not necessarily mean that the physical universe in extension transcends all limitation, nor that God is infinite in His power, goodness, and wisdom. We might mean any of these. All our doctrine demands, however, is that reality, what man faces regardless of how otherwise it may be conceived, is of such nature that no limit can be set to its essential properties. (2) Reality is everlasting, whether reality be deity, physical universe, or souls of men. Earthly life is in some kind of eternal setting. It is not essential to the doctrine that the being of which we predicate temporal endlessness and no origin in time be specified. That is the function of the forms which historically the doctrine has assumed. But man faces something, all-important in the scheme of reality, which is eternal. (3) The human will is futile. It is dominated by a power greater than itself, whether that be the will of God, or mechanical law. Under either régime man's plight is the same. Cosmological perspective demands only that our wills be ineffective, without specifying what is to take their place. (4) Mundane existence is trivial. In the sight of God, as corporeal creatures we are paltry. "I confess," says Thomas à Kempis, "that . . . I am worthy of all ridicule and contempt."[1] In the face of the astronomical world, we are absurd. Man, says Harlow Shapley, "infests"[2] the earth. How mediaeval theologians insisted upon our depravity and helplessness! But was not this only another form of the feeble insignificance to which cosmological thought since Copernicus has relegated us? Our doctrine, in its essence, states categorically that

[1] *The Imitation of Christ*, Baltimore, C. 1841 (Trans. approved by Holy See), Bk. III, Ch. 8 and Ch. 52.

[2] "Man and his Young World," in *The Nation* (American series), New York, Vol. 118, pp. 529-531 (May 7, 1924).

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human nature is utterly worthless as contrasted with its vastly more real environment, whatever this latter is assumed to be. Our doctrine stops here. To qualify further would be to interpret in favor of one or another of the historical aspects the doctrine has exhibited.

Thus cosmological perspective has always asserted infinity, eternity, determination, human worthlessness. Cosmological perspective, or, less appropriately, broad Copernicanism, accordingly transcends Copernicanism proper or the heliocentric theory. This latter specialized doctrine, nevertheless, seems to have become intimately associated with the former. We often fail to distinguish between the two. We carelessly attribute a whole philosophy to Copernicus and his scientific successors, Kepler and Galileo. Why, historically, did the synthesis occur?

Our question is not, why did any synthesis occur. Scientific theories always link with some philosophy. Our question is, why did the particular synthesis which happened occur, and not another. Why, e. g., did not the heliocentric theory connect with Berkeleyan or Kantian idealism? Nothing exists in those doctrines antagonistic to Copernicus' scientific achievement. In fact they possess a certain affinity. The heliocentric theory was a change in point of view—a transposition of centers of coordinates. How beautifully this doctrine would fit into an idealistic back-ground! Observatories might be engaged to-day in deducing various astronomical concepts from the relation of the Transcendental Unity of Apperception to the logical structure of experience. Metaphysicians might be out under the stars with telescopes, determining with mathematical precision the relations of ideas to one another and to mind. An astronomer would talk, without wincing, about the ideality of space. A metaphysician would deride scientists for being too 'me-

tempirical.' But none of this has come to pass. The heliocentric theory is inseparably bound to a philosophy which would suffer torment rather than give human mind priority or effectiveness in the scheme of things. Our question is, why this particular alignment of forces.

To regard Bruno, we have said, as a Copernican propagandist is to fail to do him justice. In fact in so far as Bruno was an agitator for the heliocentric theory he championed a more enlightened form of that doctrine than left the hands of Copernicus or Kepler. Bruno did not regard the sun as the center of the universe. The stars, he believed, also are suns, each with its own coterie of worlds. The cosmos being infinite, no sun can be said to occupy its center. But this is beside the point. Bruno was more than an agitator even for this glorified heliocentric theory. His mind was the first focus where Copernicanism proper met the ancient stream of thought which had illumined his predecessors. He championed his resulting world-view with a crusader's ardor, carrying it throughout Europe. His indefatigible labors and the dramatic circumstances of his death determined what associations the heliocentric theory was to make.

What, we may ask, has Copernicanism proper, 'telescopism,' perspective, Berkeleyan and Kantian idealism to do with necessity in Bruno's philosophy? We plan immediately to launch into our subject, but first let us prolong our digression by explaining why we digressed. Determinism was not a major issue for Bruno. Realization of the infinite extent and eternal duration of the universe carried him away. The extravagant fervor with which he expresses himself as a being in the face of immensity often hides his determinism with its attendant conviction of man's pettiness. We must be guided by knowledge of the historical relations of his thought if we are to interpret

that obscure phase accurately and emphasize it sufficiently. Internal evidence will not be wanting, but it will give us isolated fragments. To put these into a meaningful whole we must regard them in the light of the motives and tendencies under which they were formed. For this reason we have tried to suggest Bruno's place in thought.

II

Determinism follows from Bruno's idea of cause. He generally realized this, though in his ethical writings he ridiculed Calvinistic predestination[3]—a doctrine with which he should have found his views in harmony. Then why are we justified in regarding Bruno as a champion of determinism when he also advocated freedom? Because Bruno was more pre-eminently a metaphysician and cosmologist than he was a moralist. His real significance lay in a world-view. As a moral philosopher, on the other hand, he is negligible. Bruno himself says the true doctrine of determinism must be hidden from the people, because they will put it to ill use.[4]

However this may be, we can be certain of one thing. Determinism alone logically follows from Bruno's treatment of causation. Let us examine his analysis of cause as well as of principle. Cause, he says, works on an object as from without, while principle works as from within.[5] We misinterpret this analysis if we think of cause and principle as distinct from one another. Bruno criticises Aristotle for attempting to extract a knowledge of reality from verbal distinctions. But Bruno, himself, seems to fall into the same error. His relapse is, however, apparent. We must bore beneath terminology if we want

[3] *Cf.* L. Kulenbeck's *Giordano Bruno*: *Gesammelte Werke*, (6 vol.), Leipzig, 1904, Vol. III, p. 193B (note).

[4] *Cf. ibid*, Vol. III, p. 43f.

[5] *Cf.* B. Rand's *Modern Classical Philosophers*, Boston and New York, 1924, p. 5f.

to unearth his real meaning. Principle and cause are words referring not to separate entities, but to aspects of the same entity. The course a ship takes may be thought of as caused either by a pilot distinct from the ship, or by a pilot within the ship.[6] The pilot may be regarded as cause or as principle. He is either or he is both. What we call him has no bearing on his capacity, though it points out the aspect or phase of the function in which for the moment we are interested. These terms—cause and principle—have for Bruno explanatory value. They point to no objective cleavage.

Consider ignition of a meteor when it strikes earth's atmosphere. According to Bruno (assuming for purposes of exposition that he understood the conditions of meteoric phenomena) the cause of conflagration would be a force driving the meteor through space. The cause acts as if it were something exterior to the meteor, impelling it. The principle of conflagration would be this force as centered or exemplified in the meteor itself. But whether we call our force cause or principle has no bearing on the spatial position of force in relation to that of the meteor. The force is simply there, conditioning the phenomenon. We may speak of it any way we want, if our mode of expression helps us to picture what is happening. The force acts *as if* it were impelling the meteor from without. It also acts *as if* it were within the meteor, urging it along. The force, however, is the same.

Bruno sometimes uses the word 'form' to express the whole objective situation to which we refer as cause or as principle. We shall use this term henceforth to emphasize that we are treating of an objective entity rather than of a subjective abstraction.

Bruno analyzes form into (1) material, (2) constitutive, (3) efficient, (4) final. This division, like the

[6] *Cf. ibid.*, p. 10.

preceding one, points to no real objective cleavage. Otherwise Bruno's attacks on Aristotle would have no meaning. At least they would be directed only against Aristotle's particular mode of classification of causes. But we know Bruno was opposed to the whole peripatetic notion of positing multiplicity of kinds of cause. Bruno's conception of form, however, is too complicated to understand without making use of his explanatory epithets.

Material form,[7] as he treats of it, is the condition without which nothing comes into being. He sometimes calls this condition 'matter,' a misleading term. By matter he does not mean gross physical matter in the naturalistic sense. Matter, for him, signifies the least common denominator of everything. Matter is not restricted to connotations suggested by physical substrate, nor should it be interpreted even slightly in favor of such a concept. Material form is the most non-committal, most uncompromising condition possible, the condition which shall just suffice to assert existence of anything whatever. We might call the condition 'being,' except that some of us employ this word as a predicate without significance until we state a subject. Being sometimes has an *a posteriori* flavor. We may say, "The man is," or at least, "X is," and we may be unable to conceive being as having meaning absolutely alone. If being is to be synonymous with material form, we must understand being as a genuine condition having existence in its own right, and predicated of nothing except itself. We must understand being as the one subject from which every possible predicate is derived; of which every predicate, if it is to be conceived at all, must be asserted; and of which every predicate which is distinctly conceived is definitely asserted. Material form is the least common denominator of the idea of a pansy which I might conjure up in my mind, and of

[7] *Cf.* ibid., p. 18.

the planet Neptune, which at this moment occupies far out in space a definite position with regard to the sun.

A form as wide and fundamental as this material form embraces the being of vastly more than what we call actual in any ordinary sense. Such a form is also the necessary condition of existence of what we think of as possible. Being, thus, has broader significance than we usually attribute to it. Being includes possibility. With respect to being, actual men, atoms, star-clusters, steam engines, do not qualitatively differ from Zeus, Mr. Pickwick, and Utopia.

Whatever exists, therefore, does so by virtue of participation in material form. Contrariwise, whatever does not exist owes this short-coming to isolation from that form. Let Z be something inconceivable and impossible. We cannot specify Z further, because if we could Z would *ipso facto* be conceivable. Indeed the fact that we call it Z makes it partly imaginable, so that insofar it is an inadequate illustration. At any rate, we mean that whatever is impossible is so because of its being unconditioned by material form.

The crux of this doctrine is identification of actuality and possibility, or, rather, the redefining of them in such terms that their significance is taken up into a higher category[8]—being in Bruno's peculiar sense, or material form. We must forget our prejudice about any difference between these two states. Yet we must guard against the notion that they are united because Mr. Pickwick, e. g., *really* exists somewhere in the same sense as we do. Bruno does not mean this. For him the notion of a qualitative difference between the actual and the possible rests on verbal distinction. Why should I regard you as existing and a fancy as not existing? My fancy performs its

[8] *Cf.* A. Lasson's *Von der Ursache, dem Prinzip, und dem Einen* (a translation from the Italian), Leipzig, 1923, p. 72.

function in the total scheme as you do. Or why should I regard you as existing and some fancy in the mind of God as not existing?

This synthesis, we said, is the crux of Bruno's doctrine about material form. If being is oblivious to our abstraction of actuality or of possibility from a total situation, then any assertion that an existent thing might be non-existent is absurd. The precise order of the universe, so far as the bare being of each part is concerned, exhausts all possibility. What does this imply? We are compelled to conclude that whatever is, exists of necessity. We have extracted one aspect of a rigid determinism. Existence in every particular case is of such a nature that non-existence in that particular case is an impossibility. Contrariwise, whatever is non-existent is so by virtue of impossibility of existence.

Complete determinism does not follow. We suggested this when we said that we had extracted only one aspect. Vast difference holds between saying, "Isaac exists," "Isaac is tall," and "Isaac has no freedom of choice." No kind of true libertarian philosophy, however, could consistently be grafted on existential determinism. Every quality and every motion is in one sense an entity of which existence may be predicated, so that in the end Bruno's material form leaves no room for freedom of any kind.

The fact remains that when we approach determination from the point of view of being, or from that of qualified being, or from that of operations in which existent things participate, we attack the problem under different aspects. The problem objectively is one and the same, but subjectively our modes of investigation vary. Bruno tried to reach the same conclusion by arguing through various channels. His premises, in each argu-

ment, are different, but the necessity at which he arrives is the same necessity.

We should not accuse Bruno of wielding superfluous arguments unless we criticise the scientist or historian for substantiating his conclusions by fortifying one argument with another.

What were our philosopher's other arguments for necessity? We have mentioned his conception of form as embracing (1) material, (2) constitutive, (3) efficient, (4) final. Each of these kinds of form, we said, represented an abstraction from real, objective form. For Bruno these abstractions individually lay foundations for necessitarianism. By prying into the meaning of each we may extract one or another aspect of necessity. Let us examine constitutive form.

Pointing to a pebble we may say: "That exists," or, "That exists as a pebble," meaning in the latter case that as a pebble it has a certain shape, color, weight, etc., and (a Brunonian notion) a certain animatedness. Moreover, the pebble is potentially capable of acting on other things in a definite way. When we say briefly: "That exists," we refer to bare fact of existence, to material form, not caring about the particular way in which our pebble exists. But when we say: "That exists as a pebble," over and beyond referring to material form, we indicate the richness of the pebble's individual mode of existence, its qualities and potentialities, its constitutive form.

A more illuminating example is that of a human being. We point to a friend and speak of the color of his hair and eyes, his height, weight, carriage, the tone of his voice, his moods, of the way we should expect him to act under certain circumstances. All these characteristics with many more comprise his constitutive form. If we had said

simply: "He is," we should have confined our attention to mere existence.

When we say that if a body moves faster than seven miles per second it will escape from earth, we speak of constitutive form of bodies. When, with Bruno, we declare all objects are ensouled, we refer to constitutive form. In considering the strange fact that dogs are capable of giving birth only to dogs and not to cats or other animals, or that the oak exists somehow potentially in its seed we have constitutive form in mind.

Bruno's metaphor "inner artificer"[9] helps to explain this conception. Each individual existing thing has its particular mode of being as if an "inner artificer" directed its construction, spiritually and physically. The metaphor is likely to mislead in that it suggests a being distinct within every object ruling it. Bruno does not mean this. The "inner artificer" is not distinct from his abode. In fact the "inner artificer" which you house, or which I house, or which the Nebula of Orion houses, is the same, and is in no wise different from the "inner artificer" which the whole universe houses. One constitutive form alone exists, and all of it is in everything. The very use of 'in' is misleading, for non-spatial things like fancies have their "inner artificer." Things are their own constitutive form. Apple trees tend to bear apples and only apples because to do so is their constitutive form. A dyspeptic is likely to fall into a rage at the slightest provocation because that is his constitutive form.

The crux of this doctrine is a point important in all Bruno's philosophy. We have mentioned the crux in speaking of "inner artificer" which you, I, and the universe house. Constitutive form is one, pervading the entire world throughout its eternity of time and its infinity

[9] *Cf.* Rand, *op. cit.*, p. 7.

of space, and yet wholly present in every individual thing. The nature and disposition of the human being, atom, and star reflects in itself the universal plan, or comprises in itself the same plan by which the cosmos unfolds.

We may draw the conclusion we seek. Whatever is, is as it is, and 'as it is' includes the total series of qualities, potentialities, passivities, inclinations, characterizing a given entity. From the beginning every entity is whole and perfect. Whatever happens subsequently is development of the entity's own active and passive capacities. What more positive statement of determinism could we have than this?

Efficient form is the next concept from which Bruno extracts determinism. We have tried to make it clear that constitutive form does not involve actual growths, motions, developments, and exigencies of conduct, as they proceed. Constitutive form has to do only with capacities and inclinations. We may say: "Cain's character is such that we should expect him to murder Abel." On the other hand, we may say: "Cain murdered Abel." This is the point where efficient form differs from the constitutive variety. Despite Cain's character, his dispositions, emotional tendencies, he might never have actually killed Abel. But given a certain impetus over and above the sum of his bare unrealized capacities, his character will issue in the act of which his character was capable. In other words Cain will kill Abel. This final urge, resulting in achievement, is efficient form.

Here is another example, one not psychological. Constitutive form insures that a certain type seed *can* become an oak tree. Strictly this does not mean such seeds do really become oak trees. If Bruno were writing this he would draw another distinction. (1) An oak seed might not grow into a tree because of unsuitable soil and

temperature. (2) To say an oak seed becomes a tree is simply different from saying it *can* become a tree. You may state either of these situations without meaning the other. The point is, Bruno in differentiating constitutive form from efficient has this second alternative in mind. To interpret his notion correctly we must remember that primarily activities when they fail to issue from potentialities owe that deficiency not to the fact of impeding physical conditions, but to absence of efficient form.

In ordinary thought a confusion usually occurs here. When a seed fails to develop as it ought we impute the failure to natural causes. Why? Because wherever a tendency exists it inevitably results in materialization if nothing interferes. We cannot imagine a specific case where constitutive form is not accompanied by efficient. Is not Bruno's differentiation between the two types of form verbal? Yes, if you mean by verbal what does not have a counterpart in the actual universe. Bruno would grant this. As we have emphasized, these divisions of form are subjective expedients for exposition, not descriptions of real divisions in the objective world.

We have already intimated the gist of this doctrine of efficient form. Wherever a capacity exists, it must necessarily give birth to corresponding actuality, provided no other actuality intervenes.[10] But all actualities flow from capacities, so that even in case of interference our principle is not jeopardized. Instead, any instance of prevention exemplifies a result of the actualization of a group or pattern of potentialities, which pattern in the end is the entire universe.

So efficient form leads along another road to determinism. We have reached the same goal in three ways. One more path remains. It is a short and indistinct one,

[10] *Cf.* Kuhlenbeck, *op. cit.*, Vol. III, p. 37.

bearing the label 'final form.' A few remarks will suffice to characterize it. (1) Final form, or the end or aim at which all existence, development, and change in the cosmos is directed, is perfection. (2) Perfection is actualization of all possibility, fruition of all capacity. A perfect universe is one containing the maximum of wealth, variety, color, harmony, achievement, and beauty. (3) Final form works from within the cosmos. Final form is not an unattainable ideal, hanging attractively above the cosmic reach. Final form is a dream of better things entertained by the whole universe as well as by its individual contents. Final form serves as a spur to action.

In other words, our world works in such a way that each individual contributes to the universal content. Each entity draws out of the bag of possibility something which it tosses into the realm of actuality. Chance, however, plays no part in these activities. Apple trees give birth to apples because that is the end for which they exist, that is their nature. But giving birth to apples is the working out of their own nature, and is accomplished by their own efforts, by their own inherent tendencies. Similarly nebulae produce suns and planets, human beings beget human off-spring, specific ideas have specific associations Every conceivable object contributes to actuality by drawing from its own reservoir of possibility such entities as color, fragrance, symmetry, tone, goodness, brilliance.

This perpetual and universal enriching process, we said, involves no element of chance. The important point, however, is that objects in realizing their potentialities are at no stage guided or directed by outside agencies. Entities fulfill the universal purpose because it is their inherent nature to do so.

We ought not lose sight of the fact, in our discussion of transition from potentiality and possibility to actuality,

that for Bruno these states are not qualitatively distinct from one another. What is potential or possible, and what is actual, alike have being. For a potential color to develop into an actual color, thus fulfilling the universal purpose, involves only the notion of one thing having being begetting another thing having being. The situation is qualitatively the same as that of one idea associating itself with another idea. However, the transition is according to a law. There is no question of arbitrariness.

In what sense is this doctrine of final form deterministic? We find determinism in the relation between this striving (a) as it applies to the universe as a whole, and (b) as it concerns individual things. The cosmos achieves its purpose by acting through its parts. As soon as you admit that the universe acts through its parts you pave the way to determinism. For as the universe includes all possibility as well as what we ordinarily call actuality you have admitted that every activity, regardless of when or where it occurs, or in what connecton, is a particular determination of the plan by which the cosmos proceeds.

Do you remember in Newman's *Dream of Gerontius* the description of how a guardian angel's hand grasps the soul? The contact is so intimate that no portion of the soul remains untouched. In this way does final form embrace every iota of essence of each constituent in reality. Accordingly, save as embodiment of a specific purpose to add definite increments to universal wealth and variety, no entity can be said to have existence.

Bruno's doctrine of final form gives the concluding touch to his necessitarianism. It presents necessitarianism in concrete terms. Nevertheless, even this doctrine fails to afford a wholly satisfactory perspective of the place of determinism in our philosopher's world-view. For adequate insight we must discard the piecemeal method of

dissecting form into fragments which cannot exist by themselves. This method had its advantages because through dissecting we became acquainted with the machinery of Bruno's argument. But we may have failed to see the woods because of the trees. A true representation of Bruno's necessitarianism must be derived from theological premises.

Before concluding our survey, however, let us think a moment about the relations of Bruno's four formal abstractions to reality and to one another. Once we regard material or any other form as objectively factual, we overlook the extent of Bruno's reaction against Aristotle. We fail to see Bruno's conception of a solidly interconnected universe. Here are three problems: (1) To what in the real world do Bruno's four forms correspond? Surely they have some objective meaning, even granting their expository function. Otherwise they could not reveal objective truth. (2) After all, how is it conceivable that such abstractions can reveal concrete truth? (3) Do the forms bear any relation to one another?

In answer to (1) we may say the forms do correspond to something in the real world. That their function is expository means only they do not correspond to definite self-subsistent characters of reality. Science uses the same type of abstraction. When with Newton I proclaim: "Every body continues in its state of rest or uniform motion until compelled by exterior force to change that state," I refer to a situation having no objective existence. Every body acts *as though* if it were unifluenced by exterior force it would continue in its "state of rest or uniform motion." No specific body, however, has ever been known to exist in a "state of rest or uniform motion." Therefore, no example of an "exterior force" compelling a body to leave that state has ever actually been observed.

UMI

Newton's first law of motion does not describe concrete events in the real world. Newton's law abstracts from the total situation of a body moving what *would be true* if that body were allowed to move in a universe where the body and its motion alone existed. The fact remains, nevertheless, that the rest of the universe does exist, introducing disturbing factors.

Does this prevent Newton's law from being useful as an instrument in the calculation of actual objects really moving? Certainly not. As we said before, every body acts *as though* if it were uninfluenced by exterior forces it would continue in its "state of rest or uniform motion." This is important to know, and affords a basis for many deductions about instances of motion. Newton's law is not concerned with actual circumstances. Nevertheless, the law reveals one of the many principles upon the complex of which our world rises into reality.

Bruno's forms bear the same kind of relation to the universe as do scientific laws. Material or constitutive or any other of the forms would exist if the companion forms had never come into being, or if they were annihilated so that no after-effects remained. Since, however, the totality of forms exists, i. e., the world itself, forms can only be construed as rope-strands within *form,* as components whose being is conditioned by their being together. The case is similar to that of the traditional diagram of forces. A vector representing any force may be analyzed into component forces, which severally may have no actual existence, but which together comprise the force in question.

If abstract scientific generalizations like Newton's laws, or the principle of the parallelogram of forces, can furnish information about things, it is conceivable that

Bruno's abstract forms give knowledge about determinism in the universe.

Question (3) remains: Are the forms inter-related? They are related in this way. Add them together and you find that with each new addition you have a more complete statement of the nature of the world. In terms of material form the universe is bare undifferentiated being. In terms of material plus constitutive form, multiplicity and variety come into existence. Colors, sounds, odors, individual substances, potential tendencies decorate reality. But the world thus pictured is motionless and ineffective. In terms of material plus constitutive plus efficient form, reality is restricted to the abstraction seen by mechanists. Qualities, tendencies, actual motions are accounted for, but the world is a monotonously revolving set of wheels. Nowhere can aspirations, loyalties, and purposes be found. You must add final form to the preceding three forms before you get a complete expression, where teleological factors take their place.

Our answers to these three questions have this importance: they show that necessity must be an extremely intimate character of the world. Necessity must be more pervasive than any other character. No matter how thin an abstraction may be (What abstraction could be thinner than material form?) necessity is still seen to be involved. Each step from this lowest abstraction to the top, where reality is described in full concreteness, exhibits necessity.

Our answers to the three questions show something else. The process of extracting determination from the forms is almost empirical. We do not perceive necessity with the bodily eye, nor do we observe it directly through a 'rational eye.' We must think a little. But given a situation—either in nature or in our private realms of fancy—nothing more than tolerably piercing insight is

required to reveal the deterministic element. This may be inaccurate usage of the term empirical. At any rate, Bruno's method so far as the extraction of necessity from the forms is concerned is not pure science in Plato's sense. Our Philosopher, as we have interpreted him hitherto, does not attempt to deduce necessity from a first principle.

For this reason, determination as extracted from the forms or from experience retains an air of arbitrariness, a deficiency common to all results obtained by empirical methods. The law of gravitation, e. g., impresses us as a mere fact.

Bruno does not, however, leave determinism in this ultimately irrational form. He *deduces* necessity from a first principle. Some time ago we suggested a theological approach to our problem. Let us now proceed from this point of view, because Bruno's first principle is a conception of God.

III

Three remarks will suffice to elucidate Bruno's deduction proper of necessity. (1) God is one, not only insofar as unity of His mental and physical parts is concerned, but also insofar as unity of what ordinarily would be considered His intellectual attributes is concerned. God's will, power, and wisdom are one. Obviously whatever God does is co-extensive, or co-identical with what He can do and with what He knows how to do. On the other hand, whatever He knows how to do is co-identical with what He does and with what He can do. Finally, what He can do is co-identical with what He does as well as with what He knows how to do. You may arrange these three attributes to any grouping, but you will always find that God's actions and thoughts are strictly determined by the necessity of His nature.

(2) We have already mentioned the oneness of God insofar as unity of His mental and physical parts is concerned. This has particular significance because God's physical parts are the universe. Accordingly, God is co-identical with the universe, or with being in the broad sense where the universe embraces both actuality and possibility.

(3) Throughout the world, therefore, a rigid deterministic order prevails.

In our analysis of cosmological perspective we mentioned along with infinity, eternity, and determination, a fourth character—human pettiness. The stripping of effective will from man is not enough to render him "ridiculous," to use Thomas à Kempis' expression. Stars are not ridiculous, yet nobody imputes effective will to them. Why is man ridiculous? Because he is microscopic as well as ineffective. Modern astronomy teaches us that, but Bruno does so even more vividly. Bruno's universe is infinitely larger than that of modern astronomy. Accordingly, for Bruno man is infinitely smaller than say for Professor Harlow Shapley. The material universe is always small, no matter how vast its proportions. In the sight of God, the material universe is as insignificant as an individual man.

In conclusion, let us make one more remark. Bruno's necessitarianism cannot be accurately understood unless it is viewed in its proper setting, i. e., as applying to the whole of reality. The problem of whether man's will is determined is unimportant—even the problem of whether the courses of the stars are determined is unimportant. These matters shrink into trivialities when contrasted with the great law of necessity itself. For necessity is the most intimate and pervasive character of the absolute whole.

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THE UNIVERSE AND UNIVERSALS

PHILOSOPHY, it is more or less generally agreed, seeks to understand the universe. Yet whether there is such a unitary objective state of affairs as the universe, or if so in what sense there is, is a question to which often the vaguest answers are attached. That we do actually allude with apparent meaning in daily life to the sum-total of things can hardly be doubted. But it may be asked whether when we thus refer to the all-inclusive totality in the singular we may not be deluding ourselves with a merely specious object of reflection, devoid of truth and ontological significance. There are many who in one fashion or another have urged this to be the case: some claiming that "the universe" is nothing more than a name, others that it denotes some vague image in our minds, or that it is only a supreme class concept under which we subsume for convenience in our thinking the aggregate of sensuous particulars; while still others believe in the totality only as a regulative ideal, as something which we may hope for or even "make believe" to exist, but which remains at bottom a heuristic fiction bereft of being in the full sense.

Now it is the aim of the present paper to uphold the traditional or at least nominally accepted view that the universe is the subject-matter of philosophy, by maintaining not only that the universe is a possible object of investigation, but that it is the object presupposed in all

others, and possessed of ontological significance in the genuine meaning of the term. The natural place to begin such a discussion is, as it appears to us, with a consideration of the nature of universals. For however we interpret the universe, most of us will be inclined to admit that as an object it seems to differ, at least in degree of abstractness, from ordinary sensuous individuals and to share rather the character of those collective objects of generic thinking which we designate as groups, classes, or collections, and which the logician denotes by the term "universals." A universal may be provisionally explained as a unitary description which covers all of a kind; and though we may question in advance whether there can be a supreme universal equivalent to the kind of all kinds, we seem more or less obliged to assume that, if there can be knowledge of such an object inclusive of all others (*viz.* the universe), such knowledge must be of the same sort and presumably arrived at by the same means as our comprehension of universals.

Reflective experience (*i. e.* conceiving, judging, and inferring) seems to advance by laying hold of the form of objects. That is to say, whether we think, we grasp the object of thought not merely as a particular existence, but as an illustration of a general principle. For all thinking involves comparison and proceeds by the detection of similarities and differences among things. And wherever objects are compared, use is made of some identity or standard of commensurability whereby a bond of interrelation is disclosed between them. This common denominator employed by thought does not apparently have the nature of a sensuous property shared by the particulars, but has rather the appearance of a norm assumed by reflection. If I could only discover that a particular A was like or unlike a particular B by introduc-

ing a third particular *C* between the two, presumably I should never discover it, since I should have to compare *C* with *A* and *B* taken separately to make sure it was like or unlike each of them, which operation would in turn require the introduction of a further standard of comparison *D*, and so on *ad indefinitum*. The conclusion seems to follow that the reference to a standard in thinking is not to some empirical item in experience but to some determinant which we bring to the experience. Everywhere we line up and evaluate our particular objects in the light of an absolute. Although I can never see (*i. e.* sense) this piece of paper as pure white, nevertheless there is that about it that makes me know what pure white is. Similarly we judge the specific circle in the light of the idea of the perfect circle; and our act of dutifulness is performed under the inspiration of a *faciendum* which is more than any *factum*. In such instances we appeal to a determinant which is neither identical with the individual phenomenon nor with the sum-total of existent circles or acts of duty, yet which is none the less taken to be typical of all possible examples of their kinds. Thought, then, it would seem, has the power of apprehending an ideal totality from an empirical case, of seeing the one through the other, and of passing from particulars to wholes distinct from the mere collection of the parts.

But if thought really involves reference to a standard that holds for all cases, by means of which we may assess the value of our particular experiences, surely such a mode of apprehension, if any, must furnish us with knowledge of the universe. Indeed, it may even be said that the idea of the universal when fully unfolded is scarcely distinguishable from that of the universe. For although the universal is clothed with different meanings in con-

nection with different sets of objects (*e. g.* in the realm of circles it is the "perfect" circle, in that of human beauty the ideal of physical perfection, and so on), nevertheless throughout *many* appearances it continues as fundamentally *one,* since behind all diversities of application remains the unifying notion of a standard to which all things are referred. In other words, the universal, though comprehended under many shapes, everywhere expresses the principle of an ideal relation unifying things.

In addition to the claim to universality, thought assumes that it possesses ontological import. This is evident in its assertion of objective reference and in its alleged grasp of something "beyond" and "other" than itself. Practically all judging and inferring refer to an Other than the act of judging or inferring. I do not merely "think what I think" or "have an idea of an idea;" were reflection no more than this it would involve an endless series of apprehensions of mental states by mental states, and be self-refuting in its attempt to refer beyond itself. There is, however, no concrete evidence in the thinking-process of the existence of such an infinitely regressive pursuit of ideas by acts of ideation. Moreover, any skepticism as to the valid possibility of referring to an object distinct from our mental states would, if entertained, cast doubt upon the integrity of thinking altogether. For the genuineness of thought consists precisely in its power to *mean what it claims to mean;* hence any repudiation of the object of thought in general would be to challenge the validity of the process as well. It would seem then that thought on its own presuppositions, which alone render it significant, must accept the authenticity of "something more" to the world than the mere act of reflection itself.

But assuming the authenticity of the ontological ref-

erence in thinking, we have next to ask what evidence there is that this "something more" to which being is attributed is the universe. The indications which point to the universe as the ultimate object implied in reflection seem to be of three sorts: first, that involved in the discovery of the inexhaustibility of particular objects, any one of which appears when examined to implicate so vast an array of factors that no boundaries can be set to it short of the totality itself; secondly, recognition that what the mind finds in its object is uniformities, dependencies, serial connections, which hold beyond ascertainable limit; and thirdly, the conclusion that objects are imbedded not merely in an endless causal chain or linear order but in a closed, integrated system in which the totality of the constituents presumably remains constant.

First, let us consider how an indefinitely expanding reference to the universe as the sum-total of phenomena may be involved in the act of thinking. Suppose I am reflecting upon such a simple object as this book. Though my recognition of its existence implies that the object is taken to be distinct from my act of thinking about it, yet I also recognize that the book, insofar as it lends itself to being experienced, must possess a nature which is in some sense determined by the experiencer. Yet in defining the book in terms of what it can be experienced as, an indefinite number of physical and psychological factors come to be accounted part of it. Not only may the size and volume differ with the various standpoints of the observer and the differing space-time coordinates he may employ, but such aspects as color are also plainly dependent upon the sensitivity of the sensory nerve-endings of the receiving organism, the rapidity of the ether vibrations, and an impossibly intricate interplay of conditions. In brief, if the conditions of a thing enter into it as ele-

ments, no apparent boundaries can be set to its nature. Yet if this be true, and if no finite group of factors can exhaust the nature of a thing, we seem finally led to assume that nothing less than the totality of affairs or the universe itself can acquaint us with the thing in its complete essence.

Again, there is a second sense in which thought refers beyond its occasion to the totality. This is discernible in the fact that the mind always interprets its objects as an illustration of certain laws. While mere sense data are defined by what they are relative to and as such are capable of an indefinite variety of interpretations by different perceivers, the fullness of the object is not to be found in its merely qualitative character but rather in the framework of relations and universals which are somehow assumed to make it what it is. What the mind ultimately seeks and finds in its object seems to be uniformities, patterns, in brief, an "order" of nature. This ball, for instance, which I bounce will be construed by me insofar as I am scientific as an exemplification of the laws of motion and mass; while even the child who runs forward to seize it will assume that its behavior is controlled by certain fixed determinations and that, having bounced once, it may bounce again. Alike in our simplest actions and most complex thoughts we presuppose that it is possible in a given experience to pass from certain phases to certain others on the basis of some lawful identity shared among them. This tendency of thought to break down the barriers of "this" and "that," and to comprehend things in their connections as examples of relationships which hold beyond any ascertainable limit, is equivalent to the postulate of the lawfulness of nature. Nevertheless the discovery of such apparently inexhaustible dependency tells us no more than that everything has an indefinitely

inferable and referable character, and serves as a link in a chain of endless causal connections. For while it seems to tell us that nature is consistent and that what is once true must be true again, it does not tell us whether nature is coherent or show any reason why the linkage of cause and effect should be more than merely a linear series or should on the whole form a closed order or mechanical system.

Yet this third assumption, to the effect that the totality must be a more or less coherent unity of interrelated parts, appears also involved in our apprehension of objects. When I think of this book in my hand, my recognition of its genuineness implies that it is what it is; in other words, that it has a nature of its own distinct from what I or any observer determine it to be. Though man, the observer, may be the measure of things as regards their matter, apparently this is by no means the case as regards their form, which stands opposed as an *Anstoss* or foil to the mental act. We must at least affirm the subsistence of the book as something which can be referred to, classified, compared, and which acts in the same way under the same conditions. Moreover, whatever is recognized as an entity is by the same act distinguished from other entities in a purely formal way. Thus the basis for discriminating "this" object from "not-this" is not some qualitative divergence of the one from the other, but rests in the apprehension of the logical identity of each, and that each is itself in the same sense as every other. Yet in this admission that the very being of objects implies both their identity and separateness irrespective of their qualitative aspects, the assumption of coherence lies hidden. For throughout the kaleidoscopic flux of qualities and the relativity of appearances, the formal identity and plurality of entites at least remain. Laying hold upon

these basic notions as unities, science (though without any proof of actual phenomenal simplicity) has succeeded in building up elaborate atomistic pictures and hypotheses indicating ways in which phenomena in different fields can be taken as functions of one another. In other words, out of the idea of bare, almost qualityless units thought has somehow been able to construct a system of substitutions and representations, transformations and equations, which, while leaving each entity and the whole what it is, nevertheless succeeds in suggesting how these discrete, detached appearances may be correlated with each other. By degrees the notion of quality is subordinated to that of quantity. What remains the same is seen to be not the kind of stuff but the quantum of it; so that to the query "quantity of what?" the answer might well be "why, that which is everything, yet in itself nothing." In science this supposition of the constancy of the whole is expressed as the principle of the conservation of mass and energy. Only in a world governed by some such postulate of permanent relationships does it seem possible to have assurance that the balance of nature may not be upset at any moment by the intrusion of unobservable and unintelligible factors.

To assert, then, that the world is constant in quantity is to say in so many words that it is an interrelated system. For quantity means that which is made up of parts not differing from the whole relevantly save in being less. And if the parts differ fundamentally only in amount, while the amount of the whole is constant, the changes that take place within it must be transformations that return into themselves by some principle of equivalence. In other words, the elements would seem to be units which, although subjected to continuous translation and substitution in terms of one another, undergo on the whole no

formal alteration. This last assumption of our thought appears, therefore, to require not only that its object shall be consistent, but that it shall exhibit as well the characters of a closed or integrated order in which the totality of the constituents remains constant.

But having shown that a reference to the universe is involved in the act of thinking, let us ask once more whether the actual being of the universe is also implied therein? Can the mere claim to existence ever prove existence? Or the mere reference to a "Beyond" establish the reality of the Beyond referred to? In other words, is it possible from the bare abstract notion of the whole to deduce its genuineness?

Obviously, in the narrow sense all these questions must be answered negatively. Mere meaning of itself, we must agree, can never prove the reality of any particular world beyond meaning. For if once you grant that the mere *claim* to mean something is sufficient to establish the *existence* of that something, you will find yourself obliged by the same logic to admit the objective reality of every prodigious fancy for which the mind of man chooses to make the claim. On the other hand, however, it seems impossible to doubt the genuineness of the external reference of meaning altogether. If knowledge is possible at all, the integrity of meaning in a general sense (that is, our capacity upon occasion to mean what we mean and to refer to what we intend to refer to) must be accepted. Otherwise,—supposing that we never meant what we meant to mean, or never thought about what we thought we were thinking about,—reflection and experience would be self-refuting.

The situation seems to place us in a dilemma. On the one side, it has to be granted that meaning cannot prove the reality of any external realm in particular; yet, on the

other, to deny the claims of meaning altogether is to declare all deception and illusion. Escape from the enigma is only possible, apparently, through recognition that while meaning as such is unable to establish the existence of any specific object of reference, it is nevertheless not so impotent as regards the universal object-in-general. That is to say, it may here be necessary to revive the ontological argument and to accept its validity at least as regards the universe as a whole. Among philosophers Spinoza especially, it will be remembered, vigorously defended the notion that totality or "Substance," unlike all less inclusive meanings, being that in terms of which everything can be defined yet which is itself definable in terms of nothing else, "is in itself and conceived through itself;" and therefore that its essence involves existence.

That the notion of the totality or universe[1] is such a meaning seems to be so for at least two reasons. In the first place, any attempt to deny the objectivity and necessity of the total object of reference of which the thought and the thinker are both members would inevitably appeal to the authenticity of the very object denied. Any challenge of the whole must at least presuppose the genuineness of the challenge and the challenger; that is, these must themselves be construed as an authentic totality by reference to which the genuineness of *any* totality is questioned.[2] In the second place, as we have said, although the mere conceptual claim to existence is not sufficient to prove the reality of any object in particular, in the case of the universe which is the object of thought in gen-

[1] As used here the term "universe" may be taken to mean the totality of objects about which inference can take place together with all the inferences that can take place about them.

[2] While in the case of narrower concepts the inconceivability of the opposite can hardly be held to prove the existence of the object conceived of, for the reason that their essences are selected and defined by a more or less arbitrary procedure which dirempts the form from the matter,—the same cannot be said of the universe, which as the all-inclusive concept and the sum of all possible selections cannot itself be selected, or have its "what" separated from its "that."

eral it seems impossible to doubt its genuineness without doubting that of thought itself. For if all reference by thought to an external world distinct from the act of conception were illusory, knowledge would be false and meaningless; since in no case should we ever be thinking about the reality we thought we were thinking about. Clearly reflection cannot grow skeptical regarding its total object without therein repudiating the validity of the reflective process itself. In other words, the mind by its very acceptance of thought as an instrument of discovery binds itself *ab initio* to abide by the conditions of thinking, and among these recognition of the reality of the object-in-general is included as one of the conditions that makes thinking possible. We may conclude, therefore, that the universe which is the ultimate object of reference in reflection is ontologically posited by thought as having the same degree of genuineness as thought itself.

Of the various criticisms raised against the concept of the universe, and its delineation of an objective state of affairs, only the briefest notice can be taken. To allude to the universe, it is said in certain quarters, is to refer to something with no definite meaning. *Omnis determinatio negatio est;* whereas determination itself is lost when we attempt to conceive the sum-total of objects without the exclusion or negation of a single item. Even granting that the notion of a *summum genus* covers everything, it discovers nothing. Though by definition all-inclusive, as to the nature of what is included it affords no inkling. "Chaos" or nonentity so named, it is urged, would enjoy exactly as much unity and definiteness of meaning as the "universe" or "cosmos."

Yet objections of this sort overshoot the mark. It is absurd to suggest that the terms "chaos" and "cosmos" are synonyms or equivalent descriptions of the world.

In fact, an absolute chaos, if meant to stand for an aggregate of disparates totally devoid of common characters, would seem to be inconceivable. Even the most radical empiricist will usually admit this at least on empirical grounds. Supposing all entities in our world were surds or incompatibles, there could obviously be no classes, groups, or laws. Yet if nothing of the same kind occurred twice, and if nothing resembled anything else, organisms could not have adapted themselves to the environment or life survived. Furthermore, the hypothesis of absolute chaos must be discarded on logical grounds. For even the most rudimentary conception of the mind requires some element of identity, some common essence, which runs through the particulars like a thread and unites them. In the notion of the chaos, however, the common quality that is held to be shared among the particulars is precisely the *absence of all common properties,* so that the concept in paradoxically assuming as its connecting link the absence of all linkages is self-contradictory. The idea of the cosmos, on the other hand, has at least the virtue of positive consistency differentiating it from chaos, in that at any rate it allows for the possibility of a coherent unity of the aggregate.

There remains to be met, however, a more serious charge against the universe as a possible object of reflection: the imputation of self-contradiction and illegitimate totality. To refer to the universe as the sum of all possible items, it is sometimes said, implies an inconsistency, since the very notion of the whole itself constitutes an item not included in the whole referred to. Due to the singular fact that the thought-act is always one step beyond the object of thought, the universe can never be envisaged; and to imagine the contrary is simply to be self-refuting. Moreover, the attempt of thought to complete

its incompleteness generates a hopeless pursuit of totality. Thus, it is argued, any proposition which is uttered about the universe as the sum-total of facts (N) must be itself a further $(N + 1)$ fact not included in the world to which allusion was made (N). Hence a second proposition about the universe, which shall include the first proposition, becomes necessary; but this in turn calls for another, and so an endless series is generated.

Under one form or another this denial of the self-transcendent power of thought to embrace a whole while remaining a member within it has always been the chief criticism of reasoning. And in a sense the only answer to it is simply to point out that the removal of the prerogative of self-transcendence would destroy all that makes genuine thinking possible. Even the most elementary concept can hardly be evolved without making use of it, as may be shown by examples. For instance, the proper name "Croesus" we may suppose at first indicated a particular individual; later on, however, observation of similar qualities in others led to the name being taken not merely denotatively as that of a single object, but connotatively as a beginning class or name typical of a class. As the characteristics of Croesus were more and more generalized, the class represented by the name became wider, yet without ceasing to include the original Croesus as a member. As a result we have today in the concept "Croesus," —as in the case of the concept "universe,"—a class that is at the same time a member of itself. And if this constitutes a contradiction, at any rate it is one shared apparently with every term that covers both an individual and a type.

But surely the charge of contradiction, when made so sweeping as to embrace all wholes or propositions that include themselves in their own scope, loses its force.

Were this alone enough to constitute a contradiction, there would be few concepts or propositions true within the meaning of their own terms. The very statement, for example, that "There can be no true assertion concerning a proposition that is also taken as a member of itself" makes a statement covering all propositions, and hence covering itself; and is therefore on its own terms (on this view) viciously circular and false. Thus the original charge that any assertion about the universe involves a contradiction can at least be met with the *tu quoque* that the very charge of contradiction contradicts itself.

In conclusion, a more positive suggestion may be offered. A universal class or whole, we may urge implies potentially at least its embodiment in an infinity of members; that is, the universal "justice" covers an infinity of just acts, and so forth. Moreover, an infinite class or collection of objects has the peculiar property of freedom from the ordinary axiom of whole and part; with it the part need not be less than, but may be equal to or coextensive with, the whole in which it is included. Acceptance of this principle is familiar in mathematics: where the series of squares, for instance, though forming only *part* of the series of natural whole numbers, is none the less taken to be coextensive with and equal to the number of numbers in the *whole* series. Now if we assume that no contradiction is to be found in the fact that in infinite collections the part is taken as equal to and capable of embracing the whole in which it is contained, and if we further suppose that universals are infinite and inexhaustible in their applications, there would seem to be no difficulty in conceiving that universal wholes or propositions may also without inconsistency be members of themselves and included in their own scope. In other words, grant-

ing that in infinite classes the part may encompass the whole without contradiction, this principle would seem to apply equally well to universals, permitting them without imputation of inconsistency to be understood to stand both for a type (*e. g.* the universe) and for the individual falling within it.

But undeniably some among those who agree that all thinking involves reference to the universe will still find difficulty in explaining to their satisfaction how by means of universals we actually know the universe. Whereas the nature of the universal is abstract and generic, they will say, that of the universe is concrete and singular. How, then, is it possible by means of non-particular, abstract objects like universals to come to know a concrete, individual object like the universe? To answer, as we have done here, that such a synthesis is possible because it is the very nature of the one to grasp the other, and because the opposite assumption (that thought never on the whole apprehends its object) would be suicidal,—to reply thus we hope may not seem an over-easy solution. Like most ultimate answers, it is of the "this or nothing" type, aiming to force assent to a certain view by representing it as the sole alternative to complete intellectual nihilism. But even those who suspect the rigor of the dichotomy as here stated may nevertheless be led to recognize the impracticality of assuming that the universals of thought are incapable of mastering the universe. To do so would seem to fly in the face of too much experience. Does not science presuppose a world indefinitely investigable, to the number of whose laws no limits can be set? Does not almost every new acquisition of knowledge bear witness to some successful fusion of the abstract with the concrete? Surely to the modern mind which constantly beholds how seemingly adamantine facts yield to

the power of intellectual analysis, how apparently simple elements turn out to be complex, and how hitherto undreamt of worlds are brought within the circle of knowledge, any claim that the universe is in its nature inaccesible to thought must be somewhat skeptically received. Thus, while it may be true that in most particular cases the universal fails to exhaust the concrete individual because of the latter's infinity of aspects, nevertheless we must suppose that on the whole and in respect of the universe thought comprehends its object, since it too, as we have tried to show, is provided with the same element of infinity that characterizes the whole itself.

But if philosophy is concerned with the universe and universals, it is evident that its object is not one open to mere sensuous observation. On the contrary, such a subject as the universe is precluded from empirical apprehension not only because of the vast discrepancy between our sense organs and the phenomena, but for the more fundamental reason that the whole is essentially an ideal pattern and as such something more than the sum of its sensuous parts.

In this sense, both the universe and universals are non-empirical objects. Such terms as man, redness, or number, for instance, if they really convey a unitary notion of any or all of a kind, must stand for something more than short-hand descriptions of fact. While it may be agreed that historically these *generalia* are elicited from experience by separating out certain common features from the sensuous manifold, nevertheless any appeal to them as universals in thought and action presupposes that they possess an authority different in kind from that of sense experience. Thus concepts are rarely merely retrospective in their significance, but claim authority as an interpretation of the future,—a future unfettered by empirical

limits, but taken as the revelation of the determination of all possible particulars of a kind. The universal "man," for instance, must include besides milliards in the forgotten past human hosts hid deep in future time. Certainly we cannot exhaust by any device of empirical examination that indefinite sequence of humanity in the space-time order, nor can we even say that it will ever be completed. Yet we can refer to "man" significantly in the same way that we can indicate "all the rational whole numbers," even though no one will presumably have empirical acquaintance with more than the merest fraction of them. What these examples illustrate is that thought has the power to grasp its object synthetically as a logical unity, without knowing the components individually or exhaustively. Thus, in passing from the distributive to the collective significance of a group, thought exchanges the notion of a finite succession of empirical instances for that of a common essence or pervasive unity, something that can never be reached by mere counting or acquaintance with particulars. Were it not, indeed, for this element of universality in reflection enabling it to go beyond the actual cases examined, synthetic judgments, hypotheses, predictions, and intellectually controlled advance, could hardly be justified. By the same means, we may assume, the universe (though empirically inexhaustible) may none the less be grasped as a logical unity. For if thought is able to reveal the universal in the case of lesser kinds, it cannot be difficult to understand, indeed we are led by a continuous process to assume, that the universe itself as the kind of all kinds can be apprehended in the same manner.

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THE RÔLE OF MATHEMATICS IN MODERN PHYSICAL THEORY

Some Philosophical Aspects of Contemporary Science[1]

AT no time is science so much talked about and as little understood by the layman as now. It is a commonplace that science has been making tremendous progress, but it is not so generally known that physics, the most exact and developed of the experimental sciences, has, in the last decade undergone a metamorphosis. The tendency of this new physics is to operate with mathematical symbols and not with physical ideas and concepts derived from sense experience. Having transcended our ordinary perceptions, and abandoned space-time description, physics has taken on a very abstract and formal character. It is impossible to clothe the conceptions of the new physics in picturesque language, because these conceptions are predominantly concerned with relations and not with the properties of things. The calculus used is a general matrix calculus which, instead of dealing with magnitudes that can be reduced to a numerical quantity, operates with an array of terms:

A_{11}	A_{12}	A_{13}.....
A_{21}	A_{22}	A_{23}.....
A_{31}	A_{32}	A_{33}.....
.....	.....	.....

[1] Presented before Section VII of the International Congress of Mathematics, Bologna, Italy, Sept. 3-10, 1928.

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That this array is not a magnitude or a number as we know it, is shown by the fact that the commutation rule is not obeyed. If you let "x" represent a matrix, and "y" another matrix, then the multiplication of the two matrices "x" and "y" results in a matrix "xy" whose terms are not the same as those of the matrix "yx". "X" times "y" in this calculus is not equal to "y" times "x." Thus matrices are extremely difficult to deal with. A radical and highly significant advance has carried the subject not only beyond the intellectual powers of the average man, but also beyond his perception and intuition. And this at a time, ironically enough, when science is not only changing the conditions of our living, but is taking on a humanistic and cultural significance. A review in *Nature* asks "whether the intellectual gulf separating the great physical scientists from the rest of their fellows is not greater now than at any previous time in history; whether they have not usurped the place of the metaphysician." Physics by becoming a more profound and purer science has audaciously left our world of common sense and its concepts whose essential obviousness was the bond between science and the layman. Matter as we know it has disappeared. Motion isn't what we think it is, and our conceptions of space and time are too crude to deal with quantum phenomena.

Recent developments have emphasized the autonomous aspect of science. The popular view of science as something that can be verified by common sense, or as a collection of facts, is not tenable. The aim of the scientist is to understand the world; to show that the universe is thinkable and amenable to reason, not primarily to establish facts. The Edisons and the Burbanks are the discoverers of facts, the Newtons, the Maxwells and the Einsteins are the inventors of theories.

Engaged in getting-at-the truth, not in a passive con-

templation of Truth, the great scientific investigators have combined speculation with activity, and have given the world the only true metaphysics. Its concepts always changing and becoming more precise as it strikes deeper into the roots of reality, science is the most original creation of the human mind, and also the most unpredictable. Nobody knows with what fundamental entities science will operate next. Theoretical physics has already escaped from the bonds of sense, and has become non-sense for the nineteenth century materialist and rationalist who, forgetting the essential nature of science, tried to envelop it with a creed that mistook the limitations of their own minds for the limitations of science. Astronomy has been called by Bernard Shaw "a monstrous exaggeration and a fairy tale." With that healthy rationalism and common sense characteristic of the popular dramatist, he has told Henderson, his biographer, that he cannot see how "our neighbor the sun so close to us that a cloud between us can make the difference between a hot day and a cold one is 93,000,000 miles off, or even 93,000." What would Bernard Shaw make of modern physics which has given up hope of ever visualizing and reducing atomic phenomena to common sense? He would probably find himself as much out of place in the modern scientific world as Lord Kelvin or Plato.

Contemporary physics which began by discarding pictures, models, and finally common sense, finding that these could not give a correct explanation of physical phenomena, has learnt, as Prof. Max Born has well expressed it, that "the world of our imagination is narrower or more special in its logical structure than the world of physical things." Physics has taken on a more vigorous and decidedly intellectual character, shifting its emphasis from content to form. "Give me matter and motion," said

Descartes, "and I will construct the universe." "Give me relations, and I will construct matter and motion," says Eddington. What was Newtonian physics and Maxwelian electrodynamics has become unified and absorbed into the general theory of continuous manifolds of four dimensions. From a few assumptions all of physics, with the possible exception of the quantum theory, can be deduced mathematically.

At one time physics looked very much like organized common sense. It began with coordinating sense experiences of the everyday world; it was concerned with the commonplace, and for this reason its results, once they were arrived at, appeared essentially obvious. Physical theories were metaphors, and physical phenomena were visualized, intuited, and felt to be understood when we compared them to the working of common things: pulleys, inclined planes, falling bodies, levers, water waves, etc. Physics was concrete. The mind became infatuated with the simple laws which governed matter in bulk, and assumed, because it could not imagine any other possibilities regarding the laws of nature, that these statistical and mechanical laws were the fundamental laws of nature.

In this concrete physics understanding a physical process means comparing it with some familiar things, or constructing a mechanical model after it. This gives us what is essentially an "as if" knowledge based on the assumption that nature uses only those patterns of behaviour which the human mind is familiar and finds it easiest to deal with. Because concrete physics takes cognizance of our mental state, we get the feeling that it gives us more knowledge than "abstract" physics which tells us nothing concerning what things are. It is not generally realized that a knowledge of what things are is a knowledge with reference to our sense impressions

and intuitions, and therefore a knowledge which is clearly limited and inexact to the point of vagueness. Not that our senses do not know, but, to paraphrase Artemus Ward, what they do know isn't quite so.

In the beginning physical concepts were derived from touch, muscular effort, sight, bodily experiences, and innate prejudices of the mind. Mass was first defined as resistance to touch; force was muscular effort; matter was something solid and permanent, etc. One of the first achievements of modern physics was to define mass mathematically and more abstractly as the ratio between force and acceleration. By defining mass in this manner, we are enabled, to quote Prof. Cohen, "to perceive an element identical in mechanical inertia as well as in light, electricity and gravitation." We have correlated diverse phenomena, and can therefore understand each the better. The poet invents a world of relationships that are important for their psychological content, while the scientist discloses relationships that are not only significant for their intellectual form, but seem to reveal the order and unity of nature. Reality which is not the static abstraction of philosophers can be grasped only by mathematical concepts that are invariant throughout change. Reality cannot be understood in any other way. The chances are that the ultimate elements are infinitely complex and probably not definable to the human mind. But although we cannot know the content of reality (this phrase is probably meaningless) we can know its form. We can know the relations that exist, and so get at its structure. Eddington calls our attention to the fact that even if we did know, say the nature of the electron, this knowledge would be irrelevant, because it does not tell us anything about the rôle played by the electron in physical phenomena. In a recent important contribution to the new quantum theory, Heisen-

berg has shown that the very idea of an individual particle or an individual electron is an abstraction. An individual electron or particle can never be observed. Their properties are known only through their interaction with other systems.

Concrete physics becomes the victim of its own abstractions. Reality, being infinitely complex, cannot be dealt with concretely except by abstracting. When we get down to the ultimates, as we do in atomic theory, the validity of mechanics or concrete physics becomes questionable. "Mechanics is not a fundamental thing," Prof. Bridgman has said, "but in some way an effect produced by the aggregate action of a great many elementary quantum processes." Mechanical terms and notions are derived from our percepts which are ill-defined and hazy. They fit nature only approximately if at all, due to the fact that our immediate experience is with statistical effects, not with reality. Our ideas, concepts, percepts, and mental images are but processes of the mind designed to secure an adequate response to the external world. These, together with the innate prejudices of our minds, such as its intense desire for permanence in time (our invention of potential energy in physics to save the principles of conservation, or, to take a more familiar example, our almost pathological confidence in the immortality of our own soul) and its fondness for constancy in space (our concept of a regular solid)[2] have become an hindrance when thinking about atomic phenomena. When we are trying to learn the secrets of a world of things with which we have no immediate contact, we must guard against explanation in terms of the familiar. How are we going to surmount this handicap? The answer has been given by the new

[2] Probably due to the intimate connection between the brain and the eye. Mr. T. Smith (*Nature*, Feb. 18, 28) has held that vision is so constituted as to bring out the sharpness of contours of bodies.

quantum theory—by a purely mathematical formulation.

Mathematics, whose symbolism has alone remained free from our mental habits of thought, from our conditioned reflexes, our sense organs, our language, and our limited and narrow logic, is the medium through which physics expresses the order, the unity and the harmony of nature. By revealing the perfect pattern, the form of nature, mathematics reveals the beauty of creation itself. Even an amoeba becomes a divine achievement from the point of view of mathematical physics. With mathematics the mind is enabled to enter the realm of pure reason, and extract the logical essence of physical phenomena stripped of all preconceptions. Physics has plunged too deeply and has become too profound to be grasped concretely. Scientists can go further with a mechanistic theory concerning man's behaviour than they can go with mechanical explanations concerning the behaviour of the atom. Dead, dull, inert matter has been found excited, unpredictable, active, and not material. And even contradictory! The recent discovery by Davisson and Germer, of the Bell Telephone Laboratories, of the diffraction of electrons by crystals has demonstrated that particles do not always act like particles, but behave like waves. In some instances "matter" keeps together, and we have an individual particle, and in other instances it spreads out like a wave, and loses all its individuality. The waves should not be taken too seriously. All we can say is that at times a particle seems to act like an individual and at other times it loses its individuality. The waves exist only in an abstract non-Euclidian configuration space. It is the logical essence of the idea of waves, not the pictorial representation that is the basis of the new wave mechanics. Visualization of atomic phenomena has been definitely abandoned in the new quantum theory. "The origin of this new theory," to

quote Dr. Birtwistle, "was an attempt to break away from the usual kinematical and mechanical concepts, and in their place, to substitute relations between definite numbers given by experiment." The new quantum mechanics has brought out the fact that the esssence of the Bohr theory is not the picture of the electron going around a nucleus, (this is really superfluous); it is the mathematical expression, the Hamiltonian Function, which the theory starts with. This mathematical expression when subject to certain mathematical operations gives the results secured. The difference between a mathematical proposition and a physical proposition is not one of form; both are of the nature "p implies q." The difference is that in mathematics the properties of "p" are given by definition, while in physics, the properties of "p" are discovered by observation.

Dynamic models and concepts are not the permanent elements in a scientific theory—the Newtonian world picture is not a first approximation to the Einstein world picture—but the Newtonian equations are a first approximation to the Einstein equations. It is in the mathematical equations that we find the permanent structure of science, and it is mathematics that gives science its culminative aspect. The Newtonian equations constitute relations that will always be true on that particular level of experience. When we plunge more deeply, we get a more profound set of truths which have embodied in them the original truths. Einstein's gravitational equations degenerate into Newton's when the field is weak, and the velocity small. Mathematics is the connecting link between past and present science. This is not to say that pictures and models are not useful in the exact sciences. They are essential if we desire to get an intuitive grasp of the phenomena. But all pictures and models imply a

causal space-time description which has only a limited validity in quantum phenomena. Space and time are very likely only approximate conceptions. The very ingenious and picturesque assumption of the spinning electron which had such remarkable success in clearing up certain difficulties of the quantum theory is but an approximate concrete distillation of a very general and abstract principle. The internal spin of the electron may indeed be a myth. The requirement that the Schroedinger wave equation, after satisfying the general transformation theory of quantum mechanics, have the invariance demanded by the relativity theory is enough to give certain additional terms which describe some general internal motion of the electron, that, to a first approximation, may be taken for the spin of the electron.

Physics at the same time keeping in closest union with experience, now moves in the realm of pure intellect and reason. It is this duality without which contemporary physics is impossible, which is one of the supreme achievements of the human mind. Physics has become intellectualized. It is only as an intellectual system that modern physical theory succeeded in transcending space-time description which was so necessary for a rational formulation of the quantum theory. Mathematics has lifted us to an ideal world of pure thought which is not subordinated to the limitations of words and qualitative concepts. We ascend the heights where the pattern and order of the whole is revealed to us.

Many philosophers from Plato down have tried to make a similar excursion, but they did not succeed in bringing back any knowledge. Has not the scientist also lost contact with reality? We begin to grow skeptical when suddenly he makes a prediction so novel and daring, that had it not been for his flight, he would never have con-

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ceived it. The existence of quanta, the Einstein shift of the lines in the spectrum of the sun, and the diffraction of electrons by crystals are recent examples of such predictions. Here at last is an intellectual system that is giving us an understanding of the universe. And because quantum physics deals with some basic fact of the universe, it is beginning to throw light on some of the most fundamental philosophical questions of all time. Free Will, Determinism, Causality, Space and Time are beginning to receive some clarification.

Although verbalization is indeed much easier, the path to an understanding of nature, the path to a knowledge of the changeless in the midst of change is only by reason and experiment. In mathematics the mind is free to reason, and man here achieves what Strachey calls his most characteristic virtue,—complete detachment from his immediate environment. Contemporary physics, having found our concepts and language inadequate, must henceforth express itself in a medium as pure as music. Happily, in mathematics science has found a perfect fusion of form and content that even surpasses music.

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THE FORMALIZATION OF IMPLICATION

THE science of mathematical logic is slowly beginning to replace the traditional logic of Aristotle in the interest of philosophers, while mathematicians have for a longer period of time accorded it grave consideration. But so intriguing have its problems been that little attention has been paid to its historical development in its relation to the older logic of the Schools. It is my intention here to offer a brief account of one element in its development and to relate this point to traditional logic.

The emphasis upon functions and functional treatment is of the essence of mathematical logic, and the implicative function is uniquely important in its nature. Functions are, viewed logically, abstractions or forms. The view which is here to be offered is that the fundamental point of divergence of the symbolic and the Aristotelian logics is the formalized notion of the implicative relation which manifests itself in the former. In fact, a more detailed investigation might establish the proposition that the formalized notion of implication is the generic force behind the rise and development of modern formal logic.

Considering the field of logic as a whole, it is possible to divide it roughly into two sections, the one dealing with the reasons and justification of inference, the other dealing with propositions, among which the implicative relation is assumed always to obtain. The one branch leads ultimately to metaphysics; the other, to mathematics. The former is traditional logic; the latter, symbolic logic.

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Traditional logic concerns itself almost entirely with the rules and conditions of inference. For it the judgment, the fundamental logical act, is the nucleus of interest. The syllogism itself is essentially a further analysis of the judgment, for it offers at least one answer to the 'how' and 'why' of inference. Implication is the essential relation in inference, and must therefore be given grave consideration. The traditional logicians look *within the proposition for the justification of inference and implications;* the mathematical logician merely takes implication *as given.* This seemingly unimportant difference may give rise to odd consequences. No true Aristotelian would agree that the proposition 'Harvard is a university' implies that 'the square root of three is an irrational number,' or that 'Paris is the capital of England' implies 'Julius Caesar invariably read the New York Times at breakfast.' It is perhaps not necessary that symbolic logic accept these as implications either, but they have not been excluded from the major logical systems thus far developed. These queer implications result from taking implication as always holding between propositions. The only limitation placed upon the implicative function by mathematical logicians in general is that a true proposition never implies a false one.

That the interest in the grounds of implication leads to metaphysics is established by considering such outstanding representatives as Bradley and Bosanquet. On the other hand, the work of those logicians who have accepted implication as given has led them along other paths. Their endeavors have culminated either in mathematics, thus making the latter science a development of logic; into algebras which can take logic as an interpretation; or into a study of forms and systems of order as such. They have for the most part kept themselves clear of metaphysical and psychological considerations, although some

have delved into the philosophical problems of logic. These, however, are offshoots, independent of logic itself.

Like other scientists, Boole and his successors have accepted reasoning and its conditions without a preliminary investigation. They have taken propositions, whose content is unknown, and have assumed that these either imply or are implied by others. Then, by the aid of assumptions (postulates) and logical principles borrowed from traditional logic, they have built systems according to which reasoning must proceed, whatever the particular subject matter of reasoning may be. Jevons' famous "logical piano" is an extreme attempt at making reasoning mechanical.

The general method of dealing with symbols consists in substituting, in an already proven proposition or in a postulate, such symbols as will lead logically towards the desired conclusion. Then by combining the already accepted proposition (or postulate) in which the substitutions have been made, with certain other accepted propositions, the conclusion is reached. The rules for arriving at a conclusion are either the postulates or accepted principles of logic. The principle of substitution is one of these principles of logic. Sometimes the substituted symbols are simple; usually they are patterns of symbols. The propositions themselves are functional equations.[1]

The motivating principle in these deductions is a formalization of a familiar notion in traditional logic. Called the Principle of Inference, it states that if the proposition p is asserted, and the proposition p implies q is asserted, then the proposition q may be asserted independently of p. It is obvious that this principle depends upon the formalization of implication. It cannot be emphasized too strongly that this is merely a statement of a situation; not a

[1] See W. E. Johnson, *Logic*, Vol. II, pp. 123-152.

justification of inference. It is nothing more than a rule for erecting a formal system. Its justification is an entirely different investigation. As Whitehead has said,[2] "The justification of the rules of inference in any branch of mathematics is not properly a part of mathematics; it is the business of experience or philosophy. The business of mathematics is simply to follow the rule."

The origin of formalized implication is to be found, as might be expected, in traditional logic itself. Boole and those who aided in the formation of symbolic logic were not consciously attempting to improve logic. They were led into new fields almost unconsciously through interests which at their start deviated but slightly from traditional logic. Let us now consider this gradual change.

Traditional logic deals primarily with propositions or judgments. Boole's logic, on the other hand, is mainly concerned with classes and the concomitant relation of subsumption. The bridge between the two is the quantification of the predicate. It will be remembered that one of the two most important theories concerning the nature of propositions considers them statements of class inclusions or exclusions. The subject of an affirmative proposition is a class included in part or in whole in the predicate, while a negative proposition asserts that the subject is excluded in part or in whole from the class which constitutes the predicate. On the basis of this view of the nature of propositions, it is possible to quantify the predicate as well as the subject. With this done, classes can be taken in extension—which is what Boole did. The extension of the subject class is equal to or less than that of the predicate class.

What is of equal importance is that the quantification of the predicate led to the recognition that logic and

[2] *A Treatise of Universal Algebra*, p. vi.

mathematics meet on common ground. It would have been indeed remarkable if Boole, without this background, had been able to create an *algebra* of logic.

With the quantification of the predicate, the inclusion-relation obtaining between subject and predicate is transformed into a relation of equality or equivalence, and the proposition becomes the assertion of an equation, of which the subject and predicate are the terms. *Here logic and mathematics are joined.* The content or meaningfulness of the two terms of the equation can be disregarded, for the propositions are given with the quantity of the terms symbolically expressed. With the non-explicitly quantified predicate it is necessary to know the meaning of the terms in order to judge of their relative extent. Thus, knowing that there are more mortals than there are men, the proposition 'All men are mortal' can be rewritten so as to read 'All men are some mortals.' Since logic is interested in forms and not in their contents, when the proposition is given with the predicate explicitely quantified, we know all that concerns us as logicians: A=vB, v being an indefinite class symbol. Then following the rules of the game, substitutions, eliminations, etc., can be made with the result that a new order of terms and relations emerges. In short, a mathematical method can be employed. Attention must be called to the fact that it is here that the notion of *function* enters into logic. The function is one of the most powerful concepts of mathematics, nor is it less important in logic.

The science rapidly becomes more complex. Classes taken in extension and represented by algebraic symbols lend themselves readily to operations of multiplication, addition, subtraction, and division. The result is that algebraic symbols, x, y, z, ... can be treated *purely as algebraic symbols,* and with the requisite rules of opera-

tion, a system of algebra is possible which takes logic as one of its interpretations.

So rich in possible interpretations is this algebra that it is unnecessary to require a bridge between the logic of classes and that of propositions. The latter logic is merely another interpretation for the algebraic symbols composing the required pattern. The same symbols, subject to the same laws, will yield, upon interpretation, either a logic of classes or of propositions. The sign of equivalence, when between symbols interpreted as propositions, states that the *times* in which A and B are true are equal.[3]

We can now see the subtle fashion in which implication became formalized. The modern logician did not consciously set up new conditions, but unconsciously slipped into the position of mathematician, thus formalizing logic to an extent that had not been dreamed of before Descartes and Leibniz, and not much more than dreamed of by them. It did not occur to Boole that he was creating a new attitude in logic. It did not enter his mind that he might not be dealing with real implication. For him implication and subsumption are equivalent relations, and implication is a matter of subsumption. C. S. Peirce was the first, to my knowledge, to recognize that there is something queer about the relation he and his colleagues called implication.

It is upon the evidence here given, along with much more which cannot be offered in this short article, that the statement is made that Boole made use of a formalized and not a real notion of implication. Furthermore, he made no attempt to study the conditions of implication and inference, but assumed more or less naively that everyone would agree that they are founded upon subsumption. His purpose lay in deducing the consequences of propositions,

[3] One could at this point consider the formalization of truth and falsehood as it has developed in the history of mathematical logic.

in completely analyzing their implications, in determining precisely what they affirm, deny, and leave in doubt. He himself sums up his problem: "Given a set of premises expressing relations among certain elements, whether things or propositions; required explicitly the whole relation consequent among *any* of those relations under any proposed conditions, and in any proposed form."[4] As is readily seen, this problem does not include the investigation of implication itself; that relation is given. And with this given datum Boole solves his problem.

It is needless for the purposes of this article to take up each great work of mathematical logic and point out that it uses formalized implication. There is practically no difference to be found in any of the works. A variance of theory is to be expected among those who attempt to display the nature of real implication. Real definitions give rise to difference and disputation. But mathematical logicians, since they define implication nominally only, have no grounds for disagreement. The definitions of the mathematical logicians merely state *what* occurs *when* one proposition implies another. This can be decided without speculation and theory. Disagreement is to be found among symbolic logicians only in their attempts to deal with the queer cases which result from their acceptance of formalized implication. Some logicians take implication as a primitive concept, others define it. In all the works, the same conclusion is arrived at: that a true proposition implies only true propositions, and that a false proposition may imply either a true or a false proposition, but that a true proposition never implies a false proposition. This is summed up in the definition given in *Principia Mathematica,* namely that if the proposition p implies the proposition q, then either p is false or q is true. It is obvious

[4] *Laws of Thought,* p. 10.

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that this definition does not give us the real meaning of implication, if such there be. Lewis' Strict Implication is not an attempt at giving a real definition. It merely tries to get closer to real implication in order to solve the troublesome cases of a false proposition implying any proposition and of any proposition implying a true proposition.

In so far as the purpose of this article is to establish the view that the formalization of implication was the point at which symbolic logic separated from traditional logic, it is unnecessary to consider the above mentioned 'limiting' cases resulting from this formalization. Moreover much has already been written concerning them. Furthermore, these analyses do not affect logistic or systems of logic themselves, but belong rather to the class of studies concerning the philosophy of logic. All that is of interest to us in the light of the present discussion is that, while traditional logic studies the grounds and conditions of implication and inference, mathematical logic concerns itself with the problems of pure forms and with the development and analyzing of systems.

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THE NATURE OF SYSTEMS (I.)

I

THOUGH there are a number of systems of mathematics, e. g. the different geometries, algebras, etc., and quite a few of logic, e. g. the Boole-Schroeder algebra, the system of material implication and the system of strict implication, there are no complete analyses of the nature of systems themselves. Mathematicians like Pieri, Bernstein, Dickson, Church, Kline and Huntington have concerned themselves primarily with the interconnection of postulates and the reduction of the number necessary for a given system; while H. M. Sheffer, who has contributed so much to the development of logic, and who has created the new science of system-form analysis, has centered his attention on the structure of systems, rather than on the nature of their various parts and the logic of their development. So far, only in Eaton's *Symbolism and Truth* —the chapter on "Formal Deduction"—is it possible to find this subject treated at any length, and had he discussed the differences between the various kinds of systems, had he considered the problems raised by the mathematicians, and had he been critical rather than descriptive, he would have achieved a perfectly general analysis, instead of an understanding of only one kind of system.

It is the purpose of the present paper analytically to characterize all possible systems, to justify their methodology, and to attempt a solution of some of the basic

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problems that arise in dealing with them. The exposition will be essentially dogmatic, for so long as brevity is desirable, dogmatic assertions attain some virtue, even though they may reflect an undesirable attitude. In fact, there is no surety that we have attained our end completely, and there is considerable doubt as to whether all the analyses are equally satisfactory. A number of debatable propositions have been stated, firstly because they appear to advance the most hopeful conclusions, and secondly because they may prompt a wider and more intensive interest in an important though much neglected study. If the second end is achieved, either by reason of the justice or injustice of our assertions, this paper will have fulfilled most of its purpose.

In the course of the exposition it will be necessary to deal with the difference between mathematics and logic. The formalists' view will be accepted, but the reason for such a position and the interpretations put on it by Russell, Wittgenstein, Hilbert and Ramsey will, for the most part, be rejected. Their arguments will be kept in mind but will not always be explicitly characterized.

II

A system is a set of propositions, some of which (the postulates and primitive propositions) serve as premisses, from which the others (theorems) are derived by means of substitutional and inferential principles. These principles represent the form of the system;[1] the postulates and theorems, or units, its matter. These units are themselves made up of a form and a matter, which in isolation from the units are members of the base. The units and members of the base submit to various conditions, the totality

[1] The form of a system is not to be confused with Sheffer's system-form. The latter is essentially a matrix or function of systems; whereas the substitutional and inferential principles are forms which make possible the derivation of theorems even in such a matrix.

of which represent the minimal determinations of systems.

The following schema indicates the parts and conditions of any system.

(1)
Base

Elements
Correlators
  Complex and unit
  1. relations
  2. relation-functions
  3. repetitive

(2)
Matter

Postulates or
Primitive Propositions
Theorems
Definitions

(3)
Principles of Deduction
(form)

Substitution of
  1. elements
  2. complexes
  3. units
Dissolution or inference

(4)
Conditions imposed on units

Identity
Consistency
Excluded n + 1

(5)
Conditions determining Postulates

Consistency
(Independence)
(Completeness)
(Irredundancy, etc.)

III

The base is the class of those entities which are components of the units, i. e. it is the class of all the elements and correlators used in a system, and will be referred to as Class B. B is a *member-function,* whose arguments are all the entities in terms of which (the correlators) and about which (the elements), the assertions in the system are made. A system like the *Principia Mathematica* works from a base containing the propositional elements, P, Q, R, etc., and the correlators, Implies, Entails, etc.

The elements are the *significates* of the units. What

is asserted is asserted of them. They are either simple or compound, the latter being made up of simple elements. The simplicity, of course, is one of systematic assumption —those elements are simple which are not further analyzed, in terms of any other element or correlator in the base. The totality of the simple elements makes up the sub-class K of the class B. This is the class which is usually assumed as basic in mathematical postulational sets. Its introduction, I believe, is due to Huntington.

It is in terms of K that the *applicational* range of a system is determined. If K is a class of points, then the system applies to points and so on. Thus if we take the first five numbers of the *Principia* as one system, the elements are all members of the class (elementary?) propositions. This system refers to propositions and only to propositions.

Sub-class K may be further subdivided. If we take the complete logical part of the *Principia* as one system, the sub-class K has propositions and propositional functions as members. These are divided into further subclasses, P_1, P_2, and F_1, F_2, representing elementary and general propositions and classes and relations. These subclasses can be further subdivided, and these last subdivisions can be subdivided apparently without limit. Thus one could subdivide F_2 into classes of monadic, dyadic, etc. relations, and select certain groups from these classes.

IV

If the elements in a system are viewed either as concrete individuals or as representing some concrete individuals the system is *particular*. In the former case it is an existential system; in the latter it is a symbolic system referring to specific existents. If we assigned the values "this is a dog," "this is a cat" and "this is a mouse"

to the real variables P, Q, and R, the *Principia Mathematica* would deal with these three and only these three particular propositions.

A system is *general* when the elements refer ambiguously to any one of a given definite class of particulars. In both the general and particular systems, the particulars which are in one case indefinitely, and in the other, definitely represented need not be unrepeatable individuals; they may be universals. Predicates and functions are in one sense unique individuals and as such can be objects or symbols in particular systems.[2]

Every general system employs *real* variables having an unlimited possibility of values of a definite kind—definite propositions, propositional functions, numbers,

[2] A propositional function has ambiguous constituents and becomes a proposition only when these constitutents have been definitely determined. Of the symbolic representations of the propositional functions, some may be definite, while others may refer to undetermined functions. Though commonly only one kind of propositional function has been recognized, there are really three (1). $\varphi!\hat{x}$ in which both the predicate and subject are ambiguously expressed. $\varphi!\hat{x}$ says the same thing as $\psi!\hat{x}$, $\varphi!\hat{y}$, $\psi!\hat{y}$ represents the structure of any proposition with subject and predicate. (2) $\varphi\hat{x}$ which is usually called the propositional function. The predicate here is definite, but the subject is not. $\varphi\hat{x}$ does not differ from $\varphi\hat{y}$ but does differ from $\psi\hat{x}$ and $\psi\hat{y}$. (3) $\varphi!x$ represents a function with a definite subject and an ambiguous predicate. It and $\psi!x$ both express the same thing. There can be but one propositional function of the first class, so that despite the variability of its constituents, the symbol $\varphi!\hat{x}$ represents a definite propositional function. It can be read as :"something is a somewhat." φx, however, is a real variable of propositional functions which have definite predicates. It ambiguously denotes the propositional functions: "a something which is a man," "that which is hurt, etc." In every context the predicate is definite, but what this predicate is, is not determined. $\varphi!x$, on the other hand, is a real variable of propositional functions which have definite subjects: "Socrates is a something or other," "Plato is a something or other," etc., are ambiguous values of propositional functions of this form. These observations apply, *mutatis mutandis*, to propositional functions with multiple predicates and subjects. The propositional functions enter as particulars in a particular system only as a definite form or as specific propositional functions—the fact that their constituents are ambiguous, not making the propositional functions themselves ambiguous. Every proposition is a value of propositional function No. 1; specific propositional functions of the form 2 and 3 are also values of that first function, but 2 and 3 as real variables are not.

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points, etc. The connotation of the real variables, and thus of their values, in part, is expressed by the "predicate" of K. The real variables represent *any* member of such a class, and when conjoined with quantifiers represent *all* entities having the specified predicate.

A system is *formal* when the elements ambiguously connote the kind of individuals as well as ambiguously denote the specific individuals of the undetermined cl. . The "predicate" of K with any element will yield a pro· sitional function of type 1, discussed in the previ. footnote. Formal systems can be achieved by generalizi the real variables of general systems to yield *pure* va ·i- ables, whose arguments are real variables, each of which may have a completely different set of values. In this system the elements have no meaning; the system is uninterpreted. If one ignores the explanation of the meaning of the real variables in the various symbolic logics, these become uninterpreted, applying to an unspecified class of entities. They would then be formal systems, where before they were simply general.

If logic is valid for any subject-matter, and if "propositions" and "propositional functions" are only some of possible objects, then an unwarranted restriction is imposed on logic when it is restricted to these two kinds of objects alone. Either we must strain the meaning of the words, "proposition" and "propositional function" and make them serve as marks for every and any thing, or we must deny the restrictive interpretation given to the elements in logic and view them as capable of representing anything from Platonic Ideas to a Bergsonian élan vital. Even if propositions and propositional functions include the whole realm of the objects of judgment, the restriction is unwarranted, unless it be arbitrarily assumed that

the scope of logic, as a system of tautologies, must be confined to that which is judged. It is hard to see how any one can work for any length of time on the problems of logic and still restrict its confines in any way whatsoever.

A system is *abstract,* when the correlators are ambiguously denoted, though they still retain certain definite logical properties. In such a system the elements are *pure* variables and the correlators are real variables. Lewis' system of quids and quods[3], and Eaton's uninterpreted system[4] are both abstract. They neither say what they are talking about nor do they characterize their correlators, being content to assign certain 'logical' properties to their correlators, and in terms of them generate certain significant units.

Generalization of the abstract system, would mean the conversion of the real variables representing the correlators into pure variables. This would not yield a definite system, but a system-form. Prof. Sheffer has analyzed system-forms and layed down basic principles regarding them. As shall be seen, in section nine, there are various kinds of correlators, and additional rules must be added to allow for the use of his principles on the different kinds of system determined by them. System form analysis, in one sense, is a wider and more basic investigation than the present; the various systems here discussed may be said to be values of the system-form as function. But the system form is not a system; no more than a propositional function is a proposition, and no study of a function will ever completely clarify the nature of its values.

Given ambiguous symbols, five determinations are necessary in order to achieve a particular assertion. Thus if we start with aRb representing any correlator and any

[3] See *Survey of Symbolic Logic,* p. 357.

[4] See *Symbolism and Truth,* Ch. VII.

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two elements, it will be necessary to characterize the correlator as having certain logical properties like symmetry, reflexivity, etc. It would then be capable of being a unit in an abstract system. If now we characterize the correlator as representing the relation, "father," we achieve a unit of a formal system. If now we characterize, the elements a and b as representing human beings, we secure a unit of a general system. If now we identify a and b as Lancelot and Gallahad, we secure a unit of a particular system.

These five steps may have intermediaries, giving us mixed systems like general-particular, formal-general and abstract-formal. Thus, if in our last illustration we had identified a as Lancelot but did not assign a value to b we would have a unit of a general-particular system.

V

There are two kinds of "meanings" involved in connection with symbols. We shall discriminate them as *significance* and *meaning proper*. How close this is to common usage is undeterminable. Significance will be taken to be the result of a determination of interconnection of elements within a homogeneous matrix. aRb in a system represents a significant unit, with a and b as significates, correlated by R. Meaning is a property possessed by something, which even in isolation from other members of the same class, points beyond itself. It is a property possessed by a *symbol* due to its capacity to point to an element within a different matrix. Meaning is a property of symbols and depends, outside of the question of voluntary endowment, on something outside itself and its system. Significance is a property of all objects and is a function of their correlation with other members of the same system. "Cat" has meaning but no significance, "Cats eat mice" has meaning

and significance, while aRb has significance but no meaning, (providing of course that aRb, as we shall always take it, is a unit of a definite abstract or formal system.)

Significance is a characteristic of every compound and unit, and is thus to be found in every system. Meanings transcend the simples, compounds and units and thus transcend the system. Everything in a system is significant; not necessarily meaningful. All meaningful symbols can be put in a system, so that what is meaningful may achieve significance, but what is significant may be without a meaning.

Particular and general systems have elements which are both meaningful and significant. They differ only in the generality of meaning that is assigned to their elements; the correlators in both cases being definite. Formal and abstract systems have elements which are only significant. They differ in that the formal systems have meaningful correlators and abstract systems have only significant ones.

Every existent object, no doubt, is both meaningful and significant. Socrates meant philosopher and teacher to Plato. Being Plato's teacher was part of his significance. If every object is related in some way to some other object it has significance; if it stands for something, say an exemplification of a Platonic Idea, it has meaning. Meaning in this sense has little to do with judgment and knowledge.

VI

No system is possible with elements alone. They must be connected with one another. Correlators must be added. These correlators are constants in any unit or compound of any system, which is less than abstract. But even in the abstract system, they represent a constant

form, though having a variable range of meaning. The correlators are the medium through which something is said about the elements; they, in short, determine the significance of the elements.

The totality of the correlators gives the totality of connections possible to the elements in any system. These correlators have two functional uses: 1. they determine complexes or compounds, or 2. they determine units. There may be correlators which do both, such as Sheffer's stroke-function, as interpreted by Russell, in disregard to the necessary distinction between implication and entailment.[5] However, most correlators are either one or the other. The correlators in the first group bring together various simple elements so as to generate complex ones. This kind of correlator is not indispensable. Only unit correlators are absolutely necessary, for they represent the minimal form required in order to generate a significant unit. It is possible to have systems where the elements are always simple, but it is never possible to have systems where the elements are not constituents of units, and thus where there are no unit correlators. Taken simply as members of the base, uninterpreted elements are simply marks or brute facts, for they have not yet been determined by the correlators and thus have neither significance, nor by hypothesis, being uninterpreted, meaning. They are significates only when they are correlated.

The discrimination between elements and correlators is therefore unavoidable. Were all the symbols of the same standing in a system, there could be no contrast between that which is made significant and that which makes it significant. Without correlators no significance is possible; without elements there could be no functional use of the correlators. To paraphrase a famous remark: elements

[5] See G. E. Moore, "External and Internal Relations," *Arist. Soc. Proceedings*, 1919-20, p. 53, and *Monist*, Oct., 1928, p. 537.

without correlators are blind; correlators without elements are empty. Even those who have tried to forge completely uninterpreted systems have been compelled to recognize this difference. Lewis' system of meaningless strings is based on a distinction between the quids and quods, and Eaton's uninterpreted system is recognized by him to be founded on a necessary distinction between "symbols of unity" and undefined "terms."

This necessity of discrimination between significates and correlators, of matter and form, imposed on uninterpreted systems, reveals a situation of considerable consequence. If an abstract system requires such discrimination, then every formal, general and particular system must, for they are all values of some abstract one. In no system is it possible to reduce form to matter or matter to form. Over and above the elements, which in their own right may be universals or correlators, there must be unit correlators, whose presence makes possible a significant combination of these elements. An isolated element, part of no unit, and thus of no system whatsoever is merely a denotative symbol, while in view of the fact that it does have physical relations to other entities, is an abstraction, the understanding of which involves at least a tacit acceptance of some matrix of which it is a part, and thus of a correlator connecting it with other elements.

VII

It is not necessary to interpret the correlators, i. e. it is possible to have an abstract system; though it is necessary that these correlators have specific, identifiable "logical"[6] properties. An uninterpreted correlator is one which has only a functional value—it stands for a definite kind of structure, or kind of significance imposed on elements. As

[6] "Logical" properties are those definable by means of material and tautological correlators. *Cf.* section 10, below.

the abstract system is defined in terms of definite logical properties possessed by the correlators, it cannot be generalized without exchanging system for system-form. In the latter we talk about systems and any correlators; in an abstract system we have the most generalized form of a definite system with a definite kind of correlator.

In terms of the "logical" properties, the correlators are members of the sub-classes C_1, C_2, C_3, etc., of the basal class B. Each of these classes has members, all of which possess the same properties. These classes then represent *correlator-functions,* whose values are specific correlators. The selection of one of these correlators determines the *descriptive* character of the system; the selection of one of the functions, determines the *significance* of any unit. A relation like "greater than" is a specific value of the correlator-function, whose values are all those relations which are transitive, asymmetrical and irreflexive. "Less than," "descendant of," "above," etc., are all values to this same correlator-function. By determining just which one of these is being dealt with, the system is described. It is then a system of mathematics or a system of logic—in section ten, any formal system which is not a logic will be called a system of mathematics. Variation in the logical properties, i. e. substitution of different correlator-functions for any given one determines different significant units. The replacement of a symmetrical by an asymmetrical relation, which serves as a unit correlator, is tantamount then to giving a different significance to the unit.

Are correlators and elements capable of independent determination? If they are, all general systems could then apply to any entities irrespective of the meaning of their different correlators. One general system could have the same applicational range as another, and would then be distinguished from it only because its correlator was

different. It would be distinguished from a formal system only in that it would specifically isolate a class of entities, while the formal system would not.

If they are not capable of independent determination, then either the specification of the correlators prefigures the kind of elements to be dealt with, or the interpretation of the elements prefigures the specific description of the correlator. It would mean that the applicational range determined the descriptive possibilities, or that the description of the system determined its applicational possibilities. On this view we would have to say that either the fact that logic deals with propositions, propositional functions, etc., (when interpreted as a general system), determines its correlators to be implications, conjunctions, disjunctions and entailments, or the fact that it is a system using these correlators determines it to be a system about propositions and what is derived from them.

In many systems, this is the only possible view. To use a relation like "above" or "north of" is to speak of entities which are in space or in something analogous to space. It would be simple perversity, or at best a metaphor to say, "this idea is north of that idea," "this proposition is above that proposition." Wherever we have such interdetermination the systems cannot be formal; they must be general, using real variables, for a formal system uses only pure variables whose scope of values is not determined.

But logic is a formal science. It enunciates propositions that must apply to everything—to existents, fancies, the unknown, the unconsidered and the inexpressible.[7] It is indeed regrettable that Russell never recognized the wisdom of his remark that "there is a certain lordliness which the logician should preserve; he must not condescend to derive arguments from the things he sees about

[7] See Ramsey's "Foundations of Mathematics," *Proc. London Math. Soc.* Vol. 25, p. 380, seq.

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him."[8] Had he done so he would never have been so precipitately swept away by some of Wittgenstein's 'discoveries.'

If logic attempts to isolate any part of the totality of entities, and restricts the application of its propositions to only some of them, it becomes arbitrarily general, instead of being completely universal and thus formal. And further, it is impossible to determine on logical grounds alone whether there are propositions,[9] (outside of the propositions of logic) and whether they alone behave the way the *Principia Mathematica* says they do. These are questions of fact; not questions of logic. What the applicational range of logic is, is neither determinable by, nor should it be of interest to, logic. All that logic should attempt is the assertion of universal truths, i. e. propositions which are necessarily true of everything. If propositions are only some of the elements in the universe, then there is no reason why logical propositions should deal with them exclusively. Even if it be true that judgment is involved in all knowledge, and that the unit of judgment is a proposition, that is a fact which no formal logician acquires in his professional capacity. A formal logician must work within a closed system. He must not deign to use empirical data, and must not restrict the application of his subject by any arbitrary and extra-logical fiat.

The *Principia Mathematica* pretends to be a logic. If that claim be recognized, it must be taken as a formal system, referring to unspecified elements and using specific correlators. If propositions and propositional functions are not everything, then it is now a particular application of a purely formal logic. And as an applied logic, it has no virtue which a logical system that dealt with horse-

[8] *Introduction to Math. Philosophy*, p. 192.

[9] To "apply" logic to propositions is tacitly to make some ontological or existential claim for them.

chestnuts and barrooms, instead of propositions, would not have.

And not only must a logic be no less than a formal system, but it must be no more. If it be more, i. e. an abstract system, its correlators are uninterpreted, and the logical system would be undifferentiated from any others possessing the same kind of correlators. A system which used a correlator that had the same functional properties as the correlator "entailment" could not then be differentiated from a logic. We shall find shortly that there are good grounds for completely rejecting this possibility.

It is to be noted that the generalization of the correlator is only partial. The correlator-function has values which are all correlators with the same logical or functional properties. In terms of these properties, the correlator-function is constant. If generalization of the correlator-function, and thus of these specific logical properties, be attempted, the use of a specific form is transcended. We would achieve instead the concept "form in general," which is completely amorphous and could not therefore determine the elements to have significance. It would be a symbol of a form, not a form itself, no more than the word 'category' is itself a category. As the word 'function' seems to be in fashion it may be taken as the function of all correlator-functions, which in conjunction with any terms, would not represent anything significant, but would determine a significant unit when a value was given to it. Thus if the R in aRb stands for any correlator at all, (the a and b would have to stand for both simple and compound elements, for otherwise R would have been already limited as dyadic, triadic, etc.[10]), ARb would not be significant, but an ambiguous mark of a significant unit. To be a

[10] The number and character, e. g. duplication of the elements partly indicates the significance of the correlator; the correlator *determines* the significance of the elements.

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form, thus, is to be a form in particular, and a form, which even formally did not determine elements in a definite way would be no form at all.

From this it follows that it is impossible to have a system which could say everything. This is not a question of the human inability to delineate all possible correlators, but a condition imposed by the very idea of a system. A system is defined by its set of postulates; the postulates are units whose significance is a function of the elements as determined by specific unit correlators. By denying some of the logical properties of a given correlator, a contradictory correlator can be generated, which would determine the elements to have an opposite significance. Thus given a relation which is transitive, we can create one which is intransitive, and when one holds the other does not. To use both of them is to work with two systems—not one. If a system uses a correlator which has application to all possible entities it is a system which then applies to anything, but would still not be a system which says everything. It would be a formal logic or mathematics, with the variables standing for any object at all. But there can be no universal system, including all possible kinds of assertions, for the very statement of the conditions of that system will exclude others, which consider contingencies which it does not entertain. A philosophic investigation of the nature of systems cannot end therefore in the discovery of an absolutely universal system. It must be content with the discovery of the conditions involved in all systems. The best that one can find is an abstract system, having many, but not all, formal systems as values; or a system-form which gave the structure of all systems. But in neither case would we have a system which says everything about everything.

VIII[11]

Just what a system excludes and just what constitutes a contradictory or a contrary system is an important but delicate matter to decide. Does Euclidean geometry exclude or contradict the Riemannian? What is the relation between the different non-Euclidean geometries, the various logics, or between a geometry and an algebra? How is it that the same element can be correlated differently in different systems?

In abstract systems, nothing is interpreted, and all that is definite is significance; i. e. the elements are correlated in definite ways by correlators with specific "logical" properties. One abstract system differs from another only in terms of the significance accorded the elements, or what is the same thing, the logical properties of its unit correlators. As any element (which in formal and abstract systems are always represented by pure variables) may have its significance expressed by more than one abstract system, opposition between these systems is not to be found in the possibility of application. In fact any abstract systems may share the same possibilities of application. Two abstract systems are *significant contraries;* they can not have all their significant units in common, but neither of them may express a given significance. A given significance, however, should be found in some one of the totality of abstract systems. Contradiction in connection with abstract systems, accordingly, is not to be found by contrasting any two, but contrasting one with all other systems.

Any element in one abstract system may be existentially or symbolically identical with an element in another, but how and when it is a member of one, it cannot significantly be a member of the other. Contradition holds in con-

[11] After this section was written, I had the opportunity of seeing Adler's *Dialetic,* which deals in part with an analogous problem. In my review of that book in *Journal of Philosophy, XXV,* p. 501-2, I have indicated why it has been impossible to accept his interpretations.

nection with the significance of the elements, as distinguished from the foregoing contradiction of significant or abstract systems, in the sense that an element has its significance expressed in this specific, or in some other undetermined abstract system, but not in the same sense in both. There is no difficulty in supposing that a given element has its significance expressed in every possible abstract system; in fact, it seems highly probable that this is true of many elements, which are capable of many different kinds of relations and correlations. Contradiction of significant elements is not the same therefore as contradiction of abstract systems. The former can be met by supposing that an element is significantly plural, so that in terms of significance it is not one element but many. The individual as suffering one relation is significantly opposed to itself as suffering another. Whether there are kinds of significance such that no element can have them all, even when viewed as a plurality, is undeterminable in terms of abstract systems alone. The elements of abstract systems are uninterpreted; whether different abstract systems share the same elements cannot be decided, for there is no way of determining their identity.

Formal systems differ in their descriptive character. A number of them may be values of the same abstract systems, and have the same significant units, though in view of the fact that there are a number of abstract systems, there must be formal systems which differ from one another not only descriptively but significantly. One can divide the formal systems into three groups: 1. Those which have the same significant units but different descriptions, e. g. a geometry and an algebra which use correlators which are values of the same correlator-function. 2. Those which have the same description but different significant units, e. g. any two logics, geometries, etc. 3. Those which

have different descriptions and different significant units, e. g. a logic and a geometry. (As formal systems are defined in terms of their description and significance, there cannot be two formal systems which do not differ in at least one of these characters.)

1. Formal systems which have the same significant units and therefore different descriptions are descriptively opposed. If one is a logic, the other is not. Any two formal systems in this group are *descriptive contraries;* they cannot both have the same description, and neither of them may have a given description. In addition, any formal system must have one description or another, and as a system cannot have more than one (without ceasing to be definitely one formal system), the relation of *descriptive contradiction* holds between it and the totality of other formal systems, which differ from it descriptively, i. e. from all other formal systems in group one or three.

2. Formal systems which have the same description differ in their significance. Any two are either *formal subalterns* or *formal contradictories.* (They are necessarily significant contraries, for the same reason that the abstract systems of which they are values are significantly opposed, but they are such only in terms of significance, without any consideration of identity or difference in description.) They are subalterns when the postulates defining one include those defining the other. The relation of the logic of material implication to that of strict implication is one of subalternation, for the latter includes all the postulates of the former and adds some others. These two systems are such that any unit in the former will be found in the latter, and if the latter holds the former must.

They are formal contradictories, when the postulates of one are achieved by denying the postulates of the other. Formal contradiction is really verbal for it amounts to giv-

ing systems with different units the same name. The elements being uninterpreted, there is no sense in calling two systems, whose correlators have opposite logical properties, by the same descriptive term. The description of a formal system is based on the interpretation of the correlators, and the identification in description is tantamount to the use of the same interpretational meaning for different correlators. Thus if one insists on using "between" as a transitive, irreflexive and asymmetrical relation, there is only a verbal difference between it and the correlator called "between" whose properties are different. One may raise the question, of course, of verbal accuracy, and decry any but one commonly accepted use of a given correlator. "Between" it may be said is a word which means just this kind of a relation, and any other use is either linguistically perverse or actually false. To describe the result of linguistic perversity as "false" or "erroneous" is to be linguistically perverse about linguistic perversity. Only when the names of correlators are viewed as unmistakable symbols for entities having definite logical properties, instead of as arbitrary marks chosen for convenience's sake, does the question of truth and falsity come in. On such a view any two formal systems which were significantly different but descriptively the same would be related as true and false or false and false systems. The former would be descriptive contradictories; the latter would be descriptive contraries. In the former case, one would have to hold, and the other fail; in the latter, if one holds, the other must fail, though both, by hypothesis, fail.

3. When two systems differ both descriptively and significantly, in terms of a given significance and description, they are *formal contraries.* How and when one holds, the other does not, though neither may. As has been stated under 1, any system in this group is the de-

scriptive contradictory of the totality of all those systems in group 1 and 3.

Neither contrariety nor contradiction in formal systems has anything to do with the nature of elements. If the interpretation of the elements does not depend on the interpretation of the correlators, every formal system can have as many general systems as its values as there are different classes of entities. The same entities can thus also be values of different formal systems. There is no reason why an element may not be determined by a general logic, algebra, grammar, etc., for though each of these systems is descriptively contrary to another, they do not require that the elements be different in each. All that differs from system to system is the descriptive aspect of the element. It is its algebraic properties that are considered in an algebra; its logical properties in a logic, etc. It is for that reason that, except in the possible case of formal contradiction, there is no reason for supposing that if any one formal system holds the other must fail. All that can be maintained is that in terms of a given description, both cannot hold; and not that both cannot find at the same time, and with the same elements, actual exemplification.

It is only in connection with general systems that the common meaning of contradiction and contrariety has importance. Various general systems differ in applicational range; they refer to different sets of entities. If we take into consideration their description and significance, we get the following seven cases:

1. General systems which have the same significance and description but different applicational ranges are *isomorphic* systems. This is a property extensively studied by Sheffer, though understood and used before him. The validational properties of one system are identical with those of another, and the systems are intertranslatable or

systematically equivalent. Thus the "ancestor" relation when used to correlate points and numbers, gives rise to two different general systems which are, from the standpoint of validation, subject to the same rules. Such systems are nevertheless applicational contraries from the point of view of a given applicational range; it is possible that neither of them may have a given application, but if one has it the other does not.

2. General systems which have the same significance and application but different descriptions are *categorical contraries,* as well as isomorphics. They deal with different descriptive aspects of the same elements by means of correlators which are values of the same correlator function. The isomorphism which in the first case depended on the identity in the significance of elements despite their difference in meaning, now depends on the identity in the functional character of the correlators, despite the different interpretations given to them. In the first case we take different elements and show that they had the same significance; in the present case, we take the same elements with the same significance, and show that their descriptive aspects are significantly equivalent.

3. General systems which are significantly different but which have the same description and application are *general* subalterns, contraries or contradictories. The illustration given before of the relation of subalternation between the systems of strict and material implication has application now, if the range of logics be arbitrarily restricted to propositions, as a limited class of entities. Two general geometric systems illustrate general contrariety or contradiction. If one accurately characterizes space, and if space is homogeneous, the others do not, so that one only is true and all others false. Those that are false are related to one another as contraries, being such that all

fail, though if one had held the other could not. If it be granted that space itself has no structure, or geometry, contradiction between the systems cannot be based on truth or falsity but simply on significance. Here we would have to hold that any two systems, even if one is derived by denying each of the postulates of the other, are simply significant contraries, or that any given system has more than one contradictory. Difference in significance always involves two units which, except in the case where the systems are subalternately related, are incompatible and could not be members of the same system. They exclude one another, and if one holds the other must fail, though it is possible that neither might fulfill a given condition. So far we have persisted in referring to two such systems as contraries; but if one accepts any one system as arbitrarily true, any other must be false, and may be taken as its contradictory, so that any one system may have more than one contradictory, or as many contradictories as there are systems less one. Viewed from without, there is no contradiction existing between any two systems, no matter how their postulates are derived. Viewed in terms of any given system, all other systems are contradictories, no matter whether they differ because of a denial of one or many of its postulates. Accordingly the various geometries are contraries or contradictories depending on whether one of them is or is not taken to be true either arbitrarily or existentially. The inter-translation which is effected in connection with the various geometries, is not one based on isomorphism, but on the variable interpretations assigned to given elements. What is a straight in one, is not a straight in another, and it is only by finding an element in one which behaves like a straight line in another, that it is possible to achieve a translation. The various geometries differ significantly,

having different sets of postulates, and share the common generic name largely because of historic and applicational reasons. If they are viewed as abstract or formal systems, instead of as general ones, the question of their applicational compatibility could not arise, for application would not then be considered. They would remain descriptive contraries or contradictories, and the general systems which were their values could be existentially compatible. There is no reason for assuming that the different "geometries" have anything more in common than a name, or that there are unique "geometric" entities about which all of them agree to speak. Were it not for the unwarranted intrusion of questions about real space, and the equally irrelevant matter of historical origin, the whole question of the relation between the "geometrical" systems would be seen to be one of a relation between different formal systems which differed significantly and could be made to differ descriptively. They would still be systems of mathematics, but would not differ any more than a system of arithmetic and a projective geometry.

4. A general system which shared with another the same description, but differed both significantly and applicationally, would be a part with the other of a larger system, or would have only a verbal identity with the other. The former case would arise when two fragments of one system were being considered; the latter when two systems shared the same description despite the fact that they differed both in significance and in interpretation of the elements. The remarks made under No. 2 in connection with formal systems, may be extended to it.

5. General systems which have the same significance, but differ both in application and in description, are isomorphic. The recognition of isomorphism, and the subsequent identification of systems having this property, is the

first step towards foregoing the employment of general systems, for formal ones.

6. General systems which have the same application, but which differ significantly and descriptively, deal with elements which have more than one significant and descriptive aspect. The relationship of such systems to one another has been considered in connection with group No. 3 of formal systems.

7. General systems which differ in application, significance and description are contraries, generally, abstractly and formally. They may all hold in the same world at the same time, but they hold for different reasons. It will require a metaphysics to determine whether any one of these systems precludes the possibility of another, for there is nothing in any one which will decide the merits of any other.

The question of contradiction and contrariety in connection with systems is thus seen to be not a question of existential truth or falsity, but of either 1. compatibility of exemplifications, or 2. the significance or relationship of meanings. The former case arises in connection with general systems with the same application but with different significance; the other arises in connection with all systems, and does not prevent an element from being a member of more than one system, (providing it has different aspects).

The relationships of the particular systems are easily derivable from the foregoing, and will be left to the reader.

The hierarchy of systems is subject to the excluded fifth; every system is either particular, general, formal or abstract. Any two orders are hierarchic contraries; if a system is in one order it cannot be in another; though it is possible that neither of them may be in a designated order of the hierarchy. If particular-general, general-

formal, etc., systems are given a place in the hierarchy, any system of one of the n orders which make up the hierarchy will be subject to the law of the "excluded n plus 1."

IX

The necessity for the use of some such general term as "correlator" to express the form of the units and the compounds, should be obvious from the fact that not everything that determines significance is a relation and not every relation in a given context determines significance. Some of the correlators are *relation-functions*. This is a new but very important concept. The relation-function is a function of relations which have different logical properties. It must be sharply distinguished from the correlator-function, whose values are correlators all of which are descriptively different but identical in function. A correlator-function may have a relation-function as one of its values; but a relation-function can never take a correlator-function as its value. All correlators are values of some correlator-function; all relations of some relation-function. Every relation is capable of being an immediate value of some correlator-function, though not of the same correlator-function of which its relation-function is a value. We thus have correlator-functions whose values are relations, and other correlator-functions whose values are the relation-functions whose values in turn are these relations.

The creation of the relation-function seems the only way out of a logical embarrassment. It has frequently been held that some relations are sometimes symmetrical, and sometimes asymmetrical, sometimes transitive and sometimes intransitive or sometimes reflexive and sometimes irreflexive, i. e. that there are non-transitive, non-

symmetrical and/or non-reflexive relations. *These are not relations at all.* They are relation-functions having logically distinct relations as values. A relation which is symmetrical, for example, is one relation; a relation which is asymmetrical is another. No relation can be both, unless a given relation can have contradictory properties. To maintain that this is possible when the relata are taken into consideration, would be tantamount to saying that relations have their logical properties determined by the nature of what is related. This, at least, should be an unpalatable doctrine for any formal logician to swallow. And whatever the nature of the interdetermination of relations and relata it has never been maintained that the *logical* properties of the relation are modified by different relata. To hold that the logical properties of a relation depend on and change because of a modification or change in the character of what is related is as sound as to maintain that "equals" is symmetrical in England and asymmetrical in America. All that two relations, with different logical properties have in common is a name; and it is here that the use of the same descriptive term to characterize different relations results in confusion.

Consider the so-called relation "brother." When it relates brothers it is irreflexive, symmetrical and transitive; when it relates brothers to sisters it is irreflexive, asymmetrical and intransitive. It has different logical properties in both cases, and it is because of this that we must say that it cannot be the same relation in both cases. Relations would have to be held to be very accommodating furniture in a logical world, if they were to modify their logical characters in order to fit in with the sex of the children in a given family. A mythologist who held this doctrine might readily picture relations anxiously watching

the distribution of genes, so they might swoop down at birth and connect the other children in just the right way with the baby. One could write a tragedy about the relation which had to adapt itself, after a complacent life, to the birth of the only sister to fourteen brothers.

"Brother" is a relation-function with the relations "brother to a sister" and "brother to a brother" as distinct values. That these two relations have something in common is of no moment. All relations have. What is important is that they have different logical characters. The relation-function does not relate, it simply indicates that something can be related by one of its values. The various values of a relation-function may be connected so that it is possible to go from "A is a brother to a brother B" and "B is a brother to a sister C" to "A is a brother to a sister C"; and from "C is a sister to a brother B" and "B is a brother to a brother A" to "C is a sister to a brother A." Not many changes will have to be made in the logic of relations to provide for the accurate and complete treatment of relation-functions, and it seems highly advisable that these be made.

Not all relations or relation-functions are correlators. They may be the elements which are correlated; e. g. in a system which characterizes and gives the significance of dyadic relations. As relations and relation-functions are themselves universals, their correlators must be universals of a higher order, for content is always of a lower universality than form. If the relations connect universals in the first place, the correlators of these relations must be universals of a third higher order, and if these in turn are correlated, it will be by correlators of the fourth order of universality and so on.

Some of the correlators determine units in which they

themselves constitute the form. Eaton has contended that no relation can do this. If A is next to B and if C is next to D, A-next-to-B is not next to C-next-to-D, except as here by the accident of writing. It is on such grounds that Eaton maintains that no relation can relate units of which it is a determining part.

This, possibly, is how most people feel about a relation. There are nevertheless correlators which are not relation-functions which seem to have just this property. A's priority to B may be prior to C's priority to D. A's prompting of B may prompt C's prompting of D.[12] Most relations, however, do not seem to be capable of relating units of which they are the form, and it is advisable to distinguish between those which can and those which cannot relate compounds or units of which they are a part. We shall reserve the term "relation" for any form of a compound or unit having definite logical properties and which *cannot* correlate anything of which it is the form; while those correlators which constitute units or compounds whose parts are also determined by these correlators will be called *repetitive correlators.*

Repetitive correlators determine a compound element or unit which is a member of the same class to which its constituent elements belong. Eaton refers to these repetitive correlators as operations. If by this he intends to imply that they represent nothing more than physical acts, the illustrations of priority and prompting seem sufficient to indicate that this need not be so. And even in the case of what is more familiarly known as operations, such as the plus of arithmetic, there is no reason for assuming that these represent physical performances and nothing

[12] "Prompting" without qualification is a relation-function, but its values can all apply to themselves.

else. There may be no warrant for assuming that there is such a thing as plus in the world, but that possibility should not be closed arbitrarily.

Lewis' remarks[13] in this connection seem particularly pertinent. "A word of caution upon the meaning of "operation" is here necessary. It is exactly by the elimination of all peculiarly mathematical operations that a system comes to have the rigorously deductive form. For the grocer who represents his putting of one sack of sugar with another sack by 25+25=50, + is a symbol of operation. For the child who learns the multiplication table as a means to the manipulation of figures, × represents an operation, but in any rigorously deductive development of arithmetic, in Dedekind's *Was sind und was sollen die Zahlen?*, or Huntington's "Fundamental Laws of Addition and Multiplication, in Elementary Algebra," + and × are simply *relations*. An operation is something *done, performed*. The only things performed in an abstract deductive system are the *logical* operations—variables are *not* added or multiplied. . . ."

Lewis does not here consider the repetitive character of the correlators, nor the fact that even "logical operations" need not be performed, but his observation is just. Correlators, in all systems, do not operate; they are operated on. A system is static; it *is* its premisses and theorems in a definite interconnection; not the *derivation* of these theorems from these premisses. What is, is not the same as what is done in order to see what is.

Implication, disjunction, addition, subtraction, entailment, equals, etc., are repetitive correlators, or repetitive functions. "P implies Q" can be correlated by implication with "P is equivalent with Q" (i. e. P implies Q and Q

[13] In *Survey of Symbolic Logic*, p. 342.

implies P). This is equivalent to the assertion "Q implies P." Here it is evident that implication is a repetitive correlator between two compound elements, one of which is already determined by implication, and the other of which is made up of two compounds each of which is determined also by implication. Any system where the compound elements or units are elements of the same kind as the constituent elements is a homogeneous system and its correlators are repetitive.

Among the correlator-functions there may be some which have only relations as values. Such correlator-functions may readily be confused with relation-functions, which also take relations as values. Correlator-functions, however, which take relations as values, take only those relations which have the same logical properties but which differ in description. They are *descriptive correlator-functions.* Relation-functions, on the other hand, have values which have the same description; they take relations which have hitherto been given the same description but which differ in logical properties. If we use only symmetry, transitivity and reflexivity and their opposites to logically characterize relations, there are eight descriptive correlator-functions, which form a special class of correlator-functions, the values of each of which are relations determining elements to have the same significance. As isomorphism is equivalent with identity in significance, there are eight groups of isomorphic systems. Correlator-functions which have relation-functions as their values will be viewed as *relative correlator-functions;* while those which take only repetitive correlators will be described as *repetitive correlator-functions.* The different correlators and correlator-functions can now be tabulated:

Correlators	*Correlator-Functions*
1. *Relation-functions* (A is lover of B)	1. *Relative correlator-functions* (A is nonsymmetrically, etc., related to B)
2. *Relations* (A is brother to a brother B)	2. *Descriptive correlator-functions* (A is symmetrically, etc., related to B)
3. *Repetitive-correlators* ((A plus B) plus (C plus D))	3. *Repetitive correlator-functions* ((aRb)R (cRd))

X

A thorough analysis of the logical and tautological unit correlators will reveal the difference between postulates and primitive propositions. And a thorough understanding of this difference will clarify the nature of the difference between mathematics and logic. It will be easier, however, to understand the difference between the various correlators and between the postulates and primitive propositions after the two sciences have been distinguished.

Logic is the science of formal tautologies. A tautology is an exhaustive set of alternatives. A formal tautology is such a set stated in terms of pure variables. $\varphi\alpha \vee \sim\varphi\alpha$, i. e. "Socrates is a man or Socrates is not a man" is a particular tautology. $(x).\varphi x \vee (\exists x).\sim\varphi x$ "every x is a man or some x is not a man," is a general tautology.[14] It is not necessary to express such tautologies in terms of disjunctions; if we deny one of the alternatives and have the other entailed by the denial, we secure a proposition which is logically equivalent to a disjunction.

[14] $(\phi).\phi x \vee (\exists\varphi)\sim\phi x$ i. e. every predicate has this subject or some predicate hasn't" is a formal tautology.

Wittgenstein was the first to recognize the fact that logic dealt with tautologies. But as Langford pointed out "no proposition which is a tautology in the sense of Wittgenstein is a proposition certifiable on logical grounds alone,"[15] because Wittgenstein's tautologies have an existential import and this is not a characteristic of propositions which are certifiable on logical grounds alone. Only formal tautologies are of importance in logic, and these are all certifiable on logical grounds alone. The whole Wittgensteinian movement, which has culminated in the abortive second edition of the *Principia Mathematica,* is based on the attempt to derive a priori truths from matters of fact, and is best met by a little dose of Kant. "Starting with atomic propositions it is impossible to construct in the manner indicated by Wittgenstein, elementary functions of atomic propositions which are logically certifiable, because atomic propositions do not have contradictories in the required sense.[16] That all the propositions in a system like the *Principia* are tautological may be most easily seen by converting all the implications and entailments of the primitive propositions into disjunctions.

In a logical system, the tautologies which are stated as the primitive propositions serve not only as premises but express the implicative rules which allow for the derivation of the theorems; and those propositions which are stated as conclusions may in turn express implicative rules for the derivation of still other conclusions or theorems. It is because a system employs as its implicative rules the primitive propositions and theorems of logic, that it is said to be a *logical system.* This does not mean that it is a system of logical propositions (which is what we mean by a *system of logic*). It means simply that it is a system which is developed by the application of tautological rules.

[15] *Mind,* Vol. 36, p. 342.

[16] *Ibid.,* p. 345.

This is an important point. Mathematics is logically developed; it is not logic. The *Principia Mathematica* is a system which is logical and which is logically developed. The non-symbolic expositions of logic are logics but are frequently not logically developed. The identification of mathematics and logic seems to be based primarily on the confusion between a logical system and a system of logic; i. e. on the confusion between implicative rules and asserted propositions.

That logic is different from mathematics is revealed by an analysis of the kinds of assertions made by each; not by the analysis of the procedure by which they derive theorems from postulates or primitive propositions. All systems might agree in the way they derive their theorems, but this is far from making them identical. Insofar as interest in a system is restricted to the connection between the postulates and theorems, to the exclusion of any consideration of the significance of the units, no discrimination is possible between the different systems which derive their theorems tautologically. The identification of logic and mathematics is based on the partial similarity in the relationship which exists between the premisses and theorems, and on the fact that the concepts of the latter are derivable from concepts used in the former. Logic and mathematics are sister sciences just so far as they are tautological systems, developed by the employment of the same systematic rules and having no concern with the legitimacy or meaning of the assertions so connected. Logicians and mathematicians are engaged in the same task insofar as they are concerned with nothing but the systematic development of their subjects. As systems these sciences do not differ; but in terms of the kinds of significance which their premisses express there is considerable difference. If our contention that logic and mathematics

are formal sciences be correct, the question of their concepts, or more accurately, the interpretation of their variables, is largely irrelevant. Propositions, numbers, classes, relations, etc., are dealt with only in general systems, which are values of purely formal logics and mathematics.

A logical system may be described as a "logic of logic." The first use of the term "logic" refers to the character of the rules employed in the development of a system. It is taken in a sense somewhat similar to that involved in such descriptive phrases as "the logic of physics," "the logic of behavior," etc., except that it essentially refers to tautological rules of system development. The other use of the term refers to a definite kind of system (one constituted by formal tautologies) on which these systematic principles may or may not be employed. The so-called tautology in the *Principia Mathematica,* (*1.2) is no more or less a tautology than any other proposition in that system.

The confusion between the two senses of the term "logic" results from the fact that some of the principles for the deduction of the theorems are also stated as primitive propositions. It is for this reason that logic is sometimes described as the science which deduces theorems by applying logical propositions on themselves. But it is not because its primitive propositions deal with propositions or even with pure variables, not because it employs certain descriptive correlators like implication, etc., and not because its propositions allow for the derivation of theorems by being applied on themselves that a system is a system of logic, but simply and solely because its primitive propositions and theorems state formal tautologies. Otherwise any formal system would be a logic, and there would be no ground for distinguishing a logic from a schematic grammar. If logic is not to

be distinguished as a science of tautologies why have a primitive proposition like "p · ⊃ : q ⊃ p" rather than "p : ⊃ · p ⊃ q"? Why does the *Principia* use the former as a primitive proposition and not the latter? Why does it say that where an assertion sign occurs, the proposition is to be "taken as a complete assertion convicting the authors of error unless the proposition is true?"[17] As long as it is a postulate, why not postulate anything? How can there be an error in making a postulate? The only answer to these questions, I believe, is to be found in an interpretation of the authors' intention. They desired to write a logic, and they recognized that in a logic the propositions are tautologically true, even though this recognition was vague and halting. What is desired of a primitive proposition of logic is that it be a formal tautology, that it say nothing, that it be, what Langford tautologically describes as being certifiable on logical grounds alone. A tautology when it is formal has no definite content and being a tautology significantly says nothing significant. It has a definite form and can be distinguished from that which is not a tautology, but its form is such that nothing more is stated in the proposition than what is assumed. One may in fact take every primitive proposition as itself a tautological consequent of a non-symbolic proposition: "if p and —p have opposite truth-values, then if one holds the other fails." This would leave the "if . . . then" unexplained, but would nevertheless be a legitimate interpretation of a logical proposition involving p and —p. In any case, we have p and —p related in a definite way, and thus significantly, even though the result may not tell us anything new about p and —p.

The "logic of logic" refers to the systematic deduction of tautologies from tautologies. This deduction is accom-

[17] *Principia Mathematica*, p. 8 second ed.

plished by the use of tautologies. Every science that is developed in that way is systematically equivalent with every other; they are instances of the same system-function. Taken as a system, logic is an isolated instance of a general system-function; taken as a set of tautologies, it is over and above every system. As a system, it uses correlators which are values of specific correlator-functions. There may be other systems which use values of the same correlator-functions and logic as a system would be isomorphic with it. But as a set of tautologies, it determines or applies to every system, for every system can be taken as a particular value of the pure variable in a logical proposition. The whole *Principia Mathematica* is a value of the proposition: "p entails p."

When a theorem is deduced from the premisses of a logical system, be that system a logic or what not, the rule of deduction is general, and that which is deduced is particular. It is general in the sense that it applies to many systems, and the particular deduction that is made is only one application of this general rule. The rule of deduction is tautological, so that the tautology which allows for the derivation of tautologies from tautologies is distinct from and over and above these tautologies.

If the following points are kept in mind, there should be no danger of confounding a logical system with a logic. 1. Not all the rules for the development of systems, e. g. the rule of substitution, are either primitive propositions or theorems of a logic. Such a rule we shall try to show is tautological, and thus logical, but it must be assumed in order to make possible even a system of logic. A system of logic, in short, must assume principles of logic, before it can be developed. 2. Not all the postulates or theorems of a logic need be employed as principles for the development of systems, e. g. the last theorem in a logic is a

premiss to no other proposition in the system, and as there are no propositions subsequent to it is not to be used to derive other propositions from any of its predecessors. This, of course, is due to the accident of actual demonstration. What was demonstrated last could have been demonstrated earlier, had different primitive propositions been used, or a slightly different method of development taken. However, the presence of one such proposition is sufficient to indicate that a system of logic is not to be defined in terms of propositions which are applied to themselves or other propositions to achieve new theorems. 3. One can derive the totality of logical propositions without recourse to systematic principles by such a device as the table of exclusive sets.[18] Logic as a science of tautologies does not depend on the principles of logical systems. These tautologies can be stated and derived without making use of such principles. Such statements and derivations, however, presuppose logic as the science of tautological principles of inference, so that even here logic must be assumed in order to be stated.

The various primitive propositions of a logic are all materially equivalent: i. e. they are all true for the same reason. They differ, (outside of the fact that they express different forms) if at all, only in the number of pure variables with which they deal. Every proposition involving two elements, for example, in logic is a tautology, and as a tautology of two elements is the same tautology no matter how stated. The validity of one tautology does not depend on the validity of another. If it did the non-systematic development of logic would be illegitimate. As Wittgenstein has said, there is no first or last in logic taken as a science of tautologies. That we have a tautology may be most easily shown by deriving it tautologically from

[18] *Cf. Monist*, Vol. 38, p. 545.

other tautologies, but that is a question of exposition and not of validity. Every proposition in a logic, at least every one containing the same number of independent pure variables, entails and is entailed by every other. It was with rare insight that the authors of the *Principia Mathematica* designated the first propositions in their system as primitive propositions and not as postulates. Postulates are the grounds for the validity of theorems, and without these postulates there is no reason for supposing the theorems to be true. The justification for the theorem is to be found in its relation to a postulate; the justification of a logical proposition is found in itself.

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SOME RECENT BOOKS

Thomas Acquinas: His Personality and Thought. By Dr. Martin Grabmann. Trans. by Virgil Michel, O. S. B., Ph. D. Longmans, Green & Co., New York, 1928. Pp. IX + 191.

This volume makes accessible to English readers a study by an eminent German authority which was originally published in 1912 and has since been translated into Italian and French. Dr. Grabmann offers his readers scarcely more than a bare introduction to a large and difficult subject. It is a pity that so reliable and mature a scholar should not have been able to interpret his author to the modern mind with a little more enthusiasm and freedom, and with somewhat less of mere restatement of scholastic propositions. The present volume is less likely to stimulate interest in Thomistic philosophy than the more recent volume by M. Étienne Gilson with which—also recently translated into English—it invites comparison. But English literature on St. Thomas is meagre indeed; and English readers will undoubtedly be well served by this translation.

The Correspondence of Spinoza. Translated and edited with Introduction and Annotations by A. Wolf. The Dial Press, New York, 1928. Pp. 502.

What is known today to have survived of Spinoza's correspondence consists of eighty-six letters, spanning a brief period of fifteen years, 1661-1676: forty-nine letters written by Spinoza himself and thirty-seven addressed to him by a number of his contemporaries. Not only do these letters afford many interesting and significant glimpses of the complex intellectual atmosphere of that age, but they also contain much important material which scholars have long recognized to be indispensable for a correct understanding of the *Ethics*. It is the more remarkable, therefore, that hitherto there has been no complete English translation of these documents; the present volume is a real contribution to critical scholarship. Mr. Wolf has apparently not hesitated to profit by earlier translations of some of the letters, but he has just as obviously maintained a critical independence with the result that he has given us a translation that is at some points both more accurate and more readable. The "Introduction" conveniently includes concise biographical sketches of Spinoza's various correspondents, several of them little known names today. The very full annotations, which follow the text of the letters, supply a large amount of interesting and valuable historical, scientific and bibliographical material and evince the thorough and scholarly character of this latest contribution of Mr. Wolf to Spinoza research.